

Reconciling Divergent Estimates in Property Assessment Cases

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Introduction

This paper reviews a *Baraisa*,¹ *Bava Basra* (B”B) 107a, which offers three *tannaic* opinions with respect to the determination of the value of a property when three designated court-appointed experts cannot agree on a single fair price. The *Gemara* explains each of the *Baraisa*’s Tannaim and offers an opinion as to which one to follow halachically. The *Gemara* concludes with Rav Ashi saying he does not understand the *Gemara*’s recommended approach. He does not offer an opinion as to which alternative he considers best. The *rishonim* interpret the *Baraisa* in ways that raise many logical problems they have difficulty resolving. We will discuss the *Gemara* and the traditional explanations of the three *tannaic* positions, and then offer a different interpretation of the *Baraisa* that leads to results that are consistent with how a modern quantitative analyst might address the issue.

The Two Cases of the *Baraisa* and Basic Statistical Terminology

The *Baraisa* begins with the unchallenged ruling that when two of the three valuations are identical, the “one being in the minority is overruled.” We will henceforth refer to this as the “majority” rule. In statistical studies when we want to find a single value to

¹ The *Gemara* as well as an English translation is provided in the *Appendix*. When the meaning of the Aramaic word is ambiguous we transliterate it.

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represent a set of numbers, the most commonly used measures are the:

- Mean: Sum the list and divide by the number of numbers there are in the list,
- Median: The value with as many numbers in the set less than it as more than it,
- Mode: The most frequently occurring value

The *Baraisa's* decision to follow the “majority” rule, in a situation where two out of three values in a set of numbers are identical, is consistent with both the Median and Mode values.

The *Baraisa* follows with three suggested methodologies for dealing with cases where the three evaluations are different and the “majority” rule is ostensibly not applicable. It illustrates this situation for assessments of: 80, 100 and 120:

- *Tanna Kamma* (T”K): 100
- R. Eliezer b.R. Zadok (REbRZ): 90
- *Acherim*: *Osin shuma bainaihen umeshalsbin.*

Explaining *Tanna Kamma*—מציעתא

The *Gemara*² proceeds to explain the reasoning for each of these positions. It attributes T”K’s choice of 100 to מציעתא (*Metsiasa*=middle). In statistical studies the search for a designated representative for a distribution of numbers is referred to as looking for a “Measure of Central Tendency.” The Mean and Median always result in numbers that are in some way “central” to the distribution they represent. This may or may not be true for the Mode.³

² The explanations do not appear to be part of the *Baraisa* but rather the *Gemara's* analysis of each position.

³ In our case there is no unique Mode (i.e., all values appear once). Even in cases with a unique Modal value, it is possible that the Mode appears at the lower or upper end of the list of numbers and a more representative value from the center may be preferred.

The Mean and Median as “middle” representative values, date back to the Greeks. The words Mean and Median come from the Indo-European root *medhyo*, and Pythagoras and the mathematicians of his school used the word *mesotetes* for Mean. Other words drawn from the same root as Median include: mediate (in the middle of the opposing parties), medium, mitten (put your hands in the middle), meridian, and Mediterranean (the middle of the world). The *Gemara’s* מציעתא, from a historical, logical as well as linguistic standpoint, could thus conceivably be referring to either the Mean or the Median. In the 80, 100, 120 case T”K’s choice of 100 as the מציעתא is consistent with both the Mean and the Median. In most cases the Mean and Median will not be the same. For example if the assessments were 90, 100 and 130, the Median would be 100 while the Mean would be 106⅔.

In reviewing the commentators on B”B 107a we found none who suggest T”K is referring to the Mean.⁴ In fact we know of no example of any *Gemara* ever clearly employing a Mean for any purpose.⁵ Although we do not know of any commentator

⁴ The reason, in our case, not to select the Mean as the final assessment value is understandable since a single extreme assessment could have an unduly high impact on the final value.

⁵ Rambam, *Hilchos Kiddush HaChodsesh* does use the word *Emtsai* to refer to the Mean:

ו:ג משיקבץ הירח והחמה לפי חשבון זה, עד שיתקבצו פעם שנייה במהלכם האמצעי--
תשעה ועשרים יום ושתיים עשרה שעות מיום שלושים מתחילת ליל, ושבע מאות שלושה
ותשעים חלקים משעת שלוש עשרה; וזה הוא הזמן שבין כל מולד ומולד, וזה הוא חודשה של
לבנה.

The way the ancient world determined the average length of a lunar cycle was based on the fact that a Solar eclipse can only occur at the time of a lunar conjunction. Thus they noted how many days there were between two consecutive Solar eclipses and divided it by the number of lunar months that occurred during that period. The resulting value is thus the Mean lunation. However, when the *Gemara* discussed this figure it referred to this length of a lunar cycle as a minimum, not *Emtsai*, i.e.,

ר"ה כה. אמר להם ר"ג כך מקובלני מבית אבי אבא אין חדושה של לבנה פחותה מעשרים
ותשעה יום ומחצה ושני שלישי שעה וע"ג חלקים.

We should also note that there is some discussion as to whether this reference in the *Gemara* was a later insertion. See, e.g., Epstein et. al. “A 5765 Anomaly,” *Tradition*, Vol. 38, No. 3, pp. 40-59, Fall 2004.

who directly associates the *Gemara's* מציעתא with the Median, יד רמה (Yad Ramah) adopts the “majority” rule given in the opening case of the *Baraisa* to eliminate both the lower and the upper assessments,⁶ i.e.,

- Those who assess 80 and 100 agree that the value is \leq 100. Hence its value cannot exceed 100,
- Those who assess 100 and 120 agree that the value is \geq 100. Hence its value must be at least 100.

Thus, by applying the “majority” rule twice, the resulting evaluation must be 100. Although this is not the rationale behind the general selection of the Median (i.e. that it is literally the value in the middle of the list), this reasoning always results in the selection of the Median assessment value regardless if it is exactly midway between the two extreme values or not.⁷

Other than in B”B 107a, מציעתא appears many times in Talmudic works as being the middle between two extremes. For example, with respect to Mishnayos⁸ that present three cases, מציעתא is the middle case between the רישא and סיפא. Other usages of מציעתא from Targum, *Gemara* and Midrash include

Shemos 26:28

וְהַבְרִיחַ הַתֵּיכּוֹן, בְּתוֹךְ הַקְּרָשִׁים, מִבְּרֶחַ, מִן-הַקְּצֵה אֶל-הַקְּצֵה. and the middle bar in the midst of the boards, which shall pass through from end to end.

תרגום ועברא⁹ מציעתא בגו דפיא מעבר מן סיפי לסיפי

⁶ ועוד דהאי דאמר עשרים דאינון שמונים זון ואידך דקאמר מנה תרוייהו מודו לא שויא טפי ממנה ואידך דקאמר שלשים סלע דאינון ק”ך זון הוה ליה חד ואין דבריו של אחד במקום שנים והא דקאמר מנה ודקאמר שלשים תרוייהו מודו דלא שויא פחות ממנה והאי דקאמר שמונים הוה ליה חד ואין דבריו של אחד במקום שנים

⁷ Also mentioned in שלטי גיבורים as an explanation of Rashbam on B”B 107a.

⁸ One interesting example is יבמות פד. where the Mishna says there are 4 cases and proceeds to explain the first, second and fourth. When the *Gemara* then refers to the Mishna it calls the second case מציעתא.

⁹ Targum for ים התיכון is מציעתא. In Hebrew the Mediterranean is התיכון.

Yerushalmi Sanhedrin 1:1

What is the signature of the Holy One blessed be he? Rebbe Bibi in the name of R' Reuven "Emes." What is "Emes"? Resh Lakish says *aleph* is at the start of the *aleph-bais*, *mem b'emtsiasa* {middle}, *tav* at the end, to say I Hashem am first, that I did not get it from another, and other than Me there is no deity, because I have no partner, and I am the last because in the future I will not hand it over to another.

Pnei Moshe d"b Resh Lakish: The reason is that *aleph* at the beginning of the *aleph-bais*, *mem* in the middle, *tav* at the end, indicate past, present, future.¹⁰

Devarim Rabba 1:10

R' Reuven said, What is the signature of the Holy One blessed be he? "Emes." Why "Emes"? Emes has three letters. *Aleph* is the first letter, *mem emtsais*, *tav* the end, to say I am first, I am last and there is no deity other than Me.

Chidushai H'Rashah d"b mem emtsais:
That is with the doubled letters.¹¹

The *Yerushalmi* and *Devarim Rabba* citations both express the idea that the word אמת is a composite of letters from the beginning, אמצעייתא (middle) and end of the Hebrew alphabet. However, since there are 22 letters in the Hebrew alphabet, the 11th or 12th letters in the Jewish alphabet should be the middle letter, and *mem* is the

¹⁰ ירושלמי סנהדרין א:א מהו חותמו של הקדוש ברוך הוא? רבי ביבי בשם ר' ראובן אמת. מהו אמת? ... אמר ריש לקיש אל"ף רישיה דאלפא ביתא מ"ם באמצעייתה תי"ו בסופה לומר אני ה' ראשון שלא קיבלתי מאחר ומבלעדי אין אלהים שאין לי שותף ואת אחרונים אני הוא שאיני עתיד למוסרה לאחר.

פני משה ד"ה ר"ל אמר היינו טעמא דאלף רישיה דאלפא ביתא מ"ם באמצעייתה תי"ו בסופה ורמז היה הוה ויהיה.

¹¹ **דברים רבה א:י** א"ר ראובן מהו חותמו של הקדוש ברוך הוא? אמת... ולמה אמת? אמת יש בו שלש אותיות אל"ף ראשון של אותיות מ"ם אמצעיית תי"ו סופן לומר אני ראשון ואני אחרון ומבלעדי אין ה'.

יידושי הרש"ש ד"ה מ"ם אמצעייתא היינו עם אותיות הכפולות.

13th.¹² *Chidushai HaRashash* therefore explains that in addition to the 22 regular letters, there are 5 letters that are written differently when they are at the end of a word. The middle item in an ordered set of 27 items is the 14th item. Assuming that when ordering letters, the regular letter precedes the end form letter, the 14th letter is the regular מ.

All of the above examples deal with sequencing of words or cases and not with numbers, and refer to something that lies exactly in the center of two other things. Thus, it is possible that מציעתא refers to a central item exactly midway between two extreme values (we will call it the mid-range) and not the Median which may not be in the exact middle. Indeed, Rambam explains מציעתא in B”B 107a in such a way:

Three who went to make an assessment—if one says 100 and two say 200, or if one says 200 and two say 100, the individual is nullified because he is in the minority. One says manna {100}, one says 80 and one says 120 the judgment is 100. One says 100, one says 90 and one says 130, the judgment is 110. And this is the manner in which judgments are always made.¹³

¹² This assumes מציעתא means the precise middle. However, the word the *Gemara* uses for the precise middle is חציו, e.g., קידושין ל. לפיכך נקראו ראשונים סופרים שהיו סופרים כל האותיות שבתורה שהיו אומרים וא"ו דגחון חציון של אותיות של ס"ת דרש דרש חציון של תיבות והתגלה של פסוקים יכרסמנה הזיר מיער עי"ן דיער חציון של תהלים והוא רחום יכפר עון חציו דפסוקים בעי רב יוסף וא"ו דגחון מהאי גיסא או מהאי גיסא ... בעי רב יוסף והתגלה מהאי גיסא או מהאי גיסא Perhaps then מציעתא means something close to the middle but not necessarily the exact middle. Thus, although the exact middle of 22 letters are the 11th and 12th, i.e., כ and ל, the triplet of letters have to spell an existing word. אמת is such a word and since it is “a” (even though it is not “the”) central letter, it was used.

¹³ מלוה ולוה כב:יד שלושה שירדו לשום--אחד אומר במנה ושניים אומרים במאתים, או אחד אומר במאתים ושניים אומרים במנה--בטל יחיד במיעוט. אחד אומר במנה ואחד אומר בשמונים ואחד אומר במאה ועשרים, נידון במאה; אחד אומר במאה ואחד אומר בתשעים ואחד אומר במאה ושלושים, נידון במאה ועשרה. ועל דרך זו שמין ביניהן לעולם Rambam clearly implies mid-range although he does not say how he derives 110. *Shulchan Aruch Choshen Mishpat* 103:3 spells it out in detail. Meiri also explains the T”K as Rambam does but offers as an example 80, 120 and 140 (with the midrange still 110 as in Rambam’s example). Rashbam

The Median for 90, 100 and 130 is 100, but Rambam's מצעיתא is 110. His interpretation, however, leads to an apparently illogical mathematical conclusion¹⁴ in that it assigns a higher valuation for 90, 100, 130 (i.e., 110) than for 100, 100, 130 (i.e. 100, based on “majority” rule) even though all the numbers in the latter triplet are greater than or equal to those of the former. One possible solution is that the 90, 100, 130 ruling is based on *peshara* (compromise) whereas the 100, 100, 130 ruling is based on the halachic principle of “*acharei rabim*” (majority rules) and the two cases need not be mathematically consistent. Rambam's position also leaves open the question as to why the *Gemara* illustrates the dispute with 80, 100 and 120 rather than the more revealing 90, 100 and 130?

Explaining Reb Elazar ben Rav Tzadok—Ignoring the Highest Assessment

The *Gemara* next explains that REbRZ's 90 valuation is based on

- dropping the highest assessment (i.e., 120),
- assuming that 90 is the correct value, and
- attributing an equal error of 10 to both the 80 and the 100 assessors, i.e., one erred on the high side and the other on the low side.

The *Gemara* asks, why not drop the lowest value of 80 and apply the same type of reasoning to the upper two assessments which results in an assessed value of 110? The *Gemara* responds:

נקוט מיהת תרי קמאי בידך דמתורת מנה לא מפקי ליה.

Use, in any event, the first two, for they do not take {the land} out of the category of *maneh*.

mentions another case according to T”K where one of the evaluations was 28 (i.e., 112). He is, however, unclear as to:

- Whether he means the middle estimate was 112 (rather than 100) or the highest estimate was 112 (rather than 120).
- Whether the final estimation stays the same as in the 80, 100, 120 case

¹⁴ See, e.g., אולם המשפט חז”מ סימן קג, .

Rashbam's explanation (Tosafos concur) that it is a result of תפשת מרובה לא תפשת (do not take an overly large sum) is unclear. Whatever assessment is made will benefit one party at the expense of the other, i.e. a low value (which the *Gemara* is suggesting) benefits the creditor and a high value benefits the debtor. On the one hand the borrower is in possession of the field, מוחזק, and this should give him the stronger position.¹⁵ On the other hand, it is also true that he is in possession of the creditor's money. In a case where the value of the property is uncertain, it seems unclear as to who should get the advantage.

Although the *Gemara* here does not mention anything about how the arrived-at valuation is implemented, *Shulchan Aruch* (*Choshen Mishpat* 103:1) says that this number is simply an initial value that is placed on the field, which is then put up for sale at this price.¹⁶ Potential buyers are then encouraged to bid up the value of the property. If the field is ultimately sold at a value greater than its opening assessment, the new actual sales price is used to pay off the creditor. If on the other hand no one is willing to pay the initial offering price, the initial price is used to determine the creditor's payment. Thus, the estimated value the *Baraisa* is discussing is meant as a minimum value protecting the borrower's interest.¹⁷ From this perspective Rashbam's תפשת

¹⁵ Ramban indeed argues that the borrower has the upper hand for precisely this reason. He then struggles to answer why we accept the lower estimates to the borrower's detriment.

¹⁶ Although the *Gemara* discusses the estimation process here in *Bava Basra*, and the public auctioning of the property in several places (e.g., *Arachin* 21a, *Bava Metzia* 35b) the exact sequencing of the procedure and the consequences of not getting a bid equal to the property valuation amount is not definitively stated in any *Gemara* (see Gra C" M 103, S" K 3). Rambam *22:6-8 מלוה ולוה* gives the general outline of the procedure but does not specifically say what happens if the initial assessment is not met. *Maggid Mishna* explains that Rambam is consistent with the position of *Shulchan Aruch*.

¹⁷ The New York Times Thursday October 28, 2010 page 26A writes: "It's not impossible that the sale of the conservatee's house to the conservator might be in the conservatee's best interest," Ms. Scott said, "but to avoid the appearance of self-dealing, a careful conservator would get three

מרוכה לא תפשת is very reasonable. Although we want to protect the borrower from getting too low a price for his property, it is unreasonable to place this lower-limit safety net at what appears to be an inflated value.¹⁸

Explaining Acherim—משלשין

The *Gemara* continues with an explanation of the “*Meshalsbin*” position of Acherim. It says this means:

- Divide the range (in this case 40: 120-80) by three¹⁹ (i.e., $40/3 = 13\frac{1}{3}$),
- Ignore the upper most value (like REbRZ), and
- Assume that both lower assessments meant $93\frac{1}{3}$ (i.e. $80 + 13\frac{1}{2}$) but erred on either side of it.

Rashbam explains that Acherim agree with REbRZ that the two lower values both were deviating from the same number, but disagrees that the deviation is 10. Since the numbers here are greater than 80, Acherim feel the amount of deviation is greater. Rashbam offers neither a rationale for 80 being the critical number, nor a me-

appraisals by commercial realtors unrelated to him and then buy the house for the highest estimate.”

¹⁸ The fact that Ramban struggles with the positions presented in this *Gemara* and does not mention that this number is only an initial minimum bid, would seem to indicate that he would disagree with *Shulchan Aruch*'s description of the procedure.

¹⁹ משלשין appears several times in Shas and generally means to divide into 3 parts. In the following case:

מכות משנה א:ג מעידים אנו את איש פלוני, שהוא חייב מלקות ארבעים--נמצאו זוממין, לוקין שמונים: משום "לא תענה בריעך עד שקר" (שמות כ,יב), ומשום "ועשיתם לו, כאשר זמם לעשות לאחיו" (דברים יט,יט), דברי רבי מאיר. וחכמים אומרים, אין לוקין אלא ארבעים. משלשין בממון, ואין משלשין במכות. כיצד: העידוהו שהוא חייב לחברו מאתיים זוז--נמצאו זוממין, משלשין ביניהן; העידוהו שהוא חייב מלקות ארבעים--נמצאו זוממין, כל אחד ואחד לוקה ארבעים.

Rashi *Makkos* 3a offers an alternate explanation:

משלשין בממון אם שלשה עדים הן או ארבעה משלשין ביניהם בית דין נעשין שליש ביניהן להשוותם בפרעון איש לפי חלקו אי נמי האי משלשין לישנא בעלמא הוא מחלקין הפרעון בין שלשתן והוא הדין נמי אם ארבעה הן מרבעין.

thodology for determining the error amount.²⁰ Moreover, the Gemara's presentation seems to relate the error amount to the difference between the highest and lowest assessments (i.e., 40) and not the size of the assessments themselves. Thus for evaluations of 20, 40 and 60 it would appear that Acherim would decide on a compromise value of $33\frac{1}{3}$.

The Gemara explains that according to Acherim both the 80 and the 100 assessments err by $13\frac{1}{3}$ from $93\frac{1}{3}$, and that the one who valued the property at 100 really wanted to say $106\frac{2}{3}$ but thought to himself "It is enough that I have exceeded my colleague's by so much."²¹ This presentation raises two issues. The first issue is that in effect the 100 assessment is being completely ignored and not contributing towards the final evaluation. The $106\frac{2}{3}$ that we are assigning to the middle assessor is derived by dividing the two extreme assessments (i.e., 80 and 120) by 3 (hence $40/3 = 13\frac{1}{3}$). This $106\frac{2}{3}$, when averaged with the lowest assessment of 80, yields the final $93\frac{1}{3}$. Presumably anything that the middle assessor would have said between $93\frac{1}{3}$ and $106\frac{2}{3}$ would have still been assigned the value of $106\frac{2}{3}$.

The second issue is how the 100 assessor knew what his colleague was going to say. Tosafos explain that "this is the order of things that the two lower assessors speak first." We are not sure how this is operationally done. How does anyone know which assessors are going to come in with the lower assessments?²² Moreover, as explained above, having the lowest assessor go first is not sufficient since the number the Gemara says the middle assessor really wanted to say is $106\frac{2}{3}$. This number however can only be calculated after we know both the lowest and the highest assessments. Rashbam addresses the question more broadly than Tosafos:

²⁰ It is also not clear from Rashbam whether REBRZ holds that 10 is a fixed amount of error or whether he too considers it the midrange between the two lower valuations.

²¹ I.e., he does not wish to exceed the lower estimate by too much.

²² Tosafos seem to take the word קמאי in the following expression literally.
נקוט מידת תרי קמאי בידך דמתורת מנה לא מפקי ליה.

...even though we do not know the words of the first one, nevertheless, they have been discussing the matter and everyone knows what is in the heart of his friend.

Rashbam's view seems to be adopted by ביאורים משה"א *Shulchan Aruch Choshen Mishpat* 103:3 *seif katan* א, who says that the three assessors should deliberate together and discuss the matter so as to get the best possible estimate.

Just as it did with REBRZ, the *Gemara* questions Acherim's primary reliance on the 80 and 100 assessments rather than on the 100 and 120. In applying Acherim's reasoning to the 100 and 120 assessments, the *Gemara* suggests that the correct number should be $113\frac{1}{3}$ and that the lower assessor deviated from the correct number by the full $13\frac{1}{3}$ and again, the person who estimated 120 did not want to exceed the 100 by too much. Tosafos ask (and do not answer) that it is illogical that three assessments 80, 100 and 120 result in a valuation of $113\frac{1}{3}$ since in a case where there are only two assessors with assessments of 100 and 120 the value of the property would be placed at 110 (i.e., each deviated by 10 from the correct value). How could a third assessment of only 80 raise the final value from 110 to $113\frac{1}{3}$? Ramban answers that in the case of two assessors, the result would not be the average of the two. Rather, the rule of "*hamotzi mechavero*" would apply.²³

The *Gemara* concludes with two different assertions that the halacha is according to Acherim. Each assertion concludes with Rav

²³ See footnote 16. Ramban also suggest that there is no such thing as having only two assessors. This, however, does not really answer the question since Tosafos' question can be formulated in a case of three assessors who came in with values of 100, 100 and 120. In this case there is no question that the value of the property would be placed at 100 (the first case of the *Baraisa*). How then could the lower triplet of assessments of 80, 100 and 120 result in the *Gemara's* suggested value of $113\frac{1}{3}$ (i.e., $13\frac{1}{3}$ more)? This variation of Tosafos' question is exactly the same as the one we previously mentioned with respect to Rambam. The fact that Ramban feels that he has an answer to Tosafos' question, and the fact that Tosafos did not ask the question from the case of three assessors, support the position previously given that we have to differentiate between cases decided by majority rule and cases decided by *pehara*.

Ashi saying he does not understand Acherim's reasoning.²⁴ Where does this leave the halacha? As we have seen, Rambam indeed does not follow the opinion of Acherim but rather that of T"K.

In the next section we offer a different interpretation of the *Baraisa* and explain how it addresses the difficulties we have raised with the way the Rishonim understood the *Gemara's* explanation of Acherim.

A Mathematical Analysis of the *Gemara*

From a practical perspective, the classical commentators' explanations of the *Gemara* that we have until here presented are, at best, incomplete. The *Baraisa* begins with the assertion that when two of the three assessors agree on a value, this is the value to be assigned. Does the word "agree" mean identical? Suppose the assessments offered were 80, 81 and 120:

- Are 80 and 81 treated as being the same (since they are so close) and based on the "majority" rule, the final assessment is placed somewhere between 80 and 81?
- Are 80 and 81 treated as different and according to Rambam's previously mentioned position the assessment is placed at 100?

If the *Baraisa* meant that to apply the "majority" rule, the two numbers would have to be literally identical, the practical application of the halacha is very limited and gives us no guidance of what to do in the overwhelming majority of cases that are not identical. On the other hand, if the *Gemara* held that when two values are

²⁴ The exact words of Rav Ashi are

אמר רב אשי טעמא דאחרים לא ידעינן הלכתא עבדינן כוותיהו.

We do not know the reason of Acherim; shall we administer the law in accordance with their view?

All of the commentators take this as a rhetorical question, i.e., I cannot follow this ruling since it is illogical. It is, however, possible to read this in an affirmative way, i.e., we will follow this decision even though we do not understand it. The argument in favor of this reading is that if Rav Ashi meant to disagree then why did he not say what he suggests be done?

“close” they can be considered as “identical,” how do we define close? For example, if rather than 80, 100 and 120 the Gemara had offered an example of 80, 100 and 480, would we say that the 480 assessment is so out of line with the other two that it may be dropped? More extreme examples can be devised until we reach the point of saying that “identical” means “close” and that “close” is relative. This means that the *Baraisa*, in the final analysis, has really not conveyed a usable methodology for dealing with the problem.

We suggest that in fact this *Baraisa* is addressing two separate issues and that the three Tannaim cited may not necessarily be in total disagreement, i.e.,

- How to handle situations where several of the estimates are identical or close.
- How to handle situations where all of the estimates are far apart.

The *Baraisa* begins with the universally accepted “majority rule” that when two estimates are identical they alone determine the value of the property. The *Baraisa* next turns to an opposite-extreme case where no two assessments are identical and the middle assessment is equidistant from the lowest and highest assessments, and offers the position of T”K. As noted previously, although Rambam understood *metsiasa* to mean mid-range, *ד רמיה* says it is the middle assessment. Using the middle assessment in the case of:

- 90, 100 and 130 (Rambam’s case) - yields 100,
- 80, 81 and 120 - yields 81,
- 100, 750 and 800 - yields 750.

Thus according to *Yad Ramah* T”K’s methodology will never allow a valuation that is totally out of conformity with the other two (whether on the high side or the low side) to overly influence the final result. T”K does not have to deal with the issue of closeness since the choice of the middle assessment (Median) guarantees that the estimates that are closer to each other will carry the most weight

The opening rule of the *Baraisa* also drives REbRZ’s approach. However, whereas T”K meant to apply his approach in all cases, we suggest that REbRZ is limiting his suggestion to a case where none

of the estimates can be considered close, e.g., they are all equidistant from each other. Rather than applying the opening majority rule twice (as T”K does), he prefers to view two of the three assessments as being the same (even though they are not). The two he chooses to use are the two lower ones and thus he adjusts the values of the property to the midpoint of these two numbers. The *Gemara* challenges this by saying that we can carry out this process equally well with the upper two assessments which would result in an assessment of 110. As we explained in a previous section, the *Gemara* responds:

נקוט מיהת תרי קמאי בידך דמתורת מנה לא מפקי ליה

I.e., if an assessment is to be discarded it is more reasonable that it be the highest one. What REbRZ’s position would be in a situation where the 3 values are not equally spaced and where the middle value is closer to the upper value, is not addressed by him. We suggest that this asymmetrical case (i.e., where the middle assessment is not equidistant from the highest and lowest assessments) is what Acherim are primarily addressing in the final case of the *Baraisa*.

In dealing with the asymmetrical case, Acherim reject T”K’s approach of always using the middle assessment, and instead offer a methodology to first measure “closeness” and then decide what to do. Their recommendation is to divide the range of the values by three and decompose the range of numbers into three intervals. In the 80, 100, 120 case, the intervals are thus 80–93 $\frac{1}{3}$, 93 $\frac{1}{3}$ –106 $\frac{2}{3}$, 106 $\frac{2}{3}$ –120. Trisecting the range in this way always results in the highest and lowest valuations falling into the two distinct end intervals. The “middle” assessment may however fall into any of the three intervals. For example, in Rambam’s case of 90, 100 and 130, the three intervals are 90–103 $\frac{1}{3}$, 103 $\frac{1}{3}$ –116 $\frac{2}{3}$ and 116 $\frac{2}{3}$ –130, and the middle assessment falls in the same third as the lower assessment. In this case Acherim would consider the 2 lower assessments to be the same, and using the “majority” rule would eliminate the highest value from consideration. Thus, unlike Rambam’s conclusion that the valuation is 110, Acherim put the final valuation between 90 and 100 and presumably would choose exactly 95 as REbRZ would. In a case of 80, 110 and 120, on the other hand, Acherim would part ways with both TK and REbRZ (since the two higher assessments

are in the highest of the three intervals, $106\frac{2}{3}$ –120) and would give the final assessment as 115 (i.e., midpoint of 110, 120).

To support our position, that Acherim are dealing with something other than the 80, 100 and 120 case, note that they never mention $93\frac{1}{3}$, nor offer any number, nor rejects outright either of the specific values offered by T”K and REbRZ. Acherim are merely trying to define when we consider two assessments to be the same and allow us to apply the “majority” rule in order to eliminate the third assessment.²⁵

What will Acherim hold in a case where all assessments fall in different intervals, e.g., the symmetrical case of 80, 100 and 120? We offer the possibility that this is the case the *Gemara* is addressing when it places Acherim’s evaluation at $93\frac{1}{3}$, i.e., the upper end of the lowest of the three intervals. Thus,

- unlike Rashbam who attributes Acherim’s disagreement with REbRZ to a desire to give more leeway for error when all the assessments exceed 80, and
- unlike the *Gemara*’s explanation that the middle assessor modified his more extreme intentions because of knowledge of what the other assessors were going to say,

the choice of $93\frac{1}{3}$ is chosen simply because it is the high point of the lowest *meshulashin* interval. This allows Acherim, like REbRZ, to give more credibility to the lower two assessments while not totally ignoring the 120 assessment²⁶ (as REbRZ does).

²⁵ Is Acherim’s choice of division by 3 arbitrary, or related to the fact that there are three assessors? Rambam seems to say that there are always 3 assessors:

מלוה ולוה כב:טו ואחר שכותבין הטירפא לטורף, מורידין שלושה בקיאין לאותה השדה, ושמין לו ממנה כשיעור חובו כפי מה שראוי לו מן הקרן, וחצי השבח כמו שביארנו.

However, Rashbam says three is not the designated number of assessors but the minimum:

ד”ה שלושה שירדו לשום... ואין שימת ב”ד פחות משלשה

If so, had *Bais Din* appointed 4 assessors would Acherim suggest division by 4 or still stick with 3? As we will explain at the end of this section, division by 3 parallels the “3 sigma rule” used in modern statistics.

²⁶ I.e., the range of the lowest interval is directly related to the 120 assessment.

From a modern statistical perspective, Acherim's choice of $93\frac{1}{3}$ in the symmetrical case is appealing for another reason. In studying distributions of numbers, the "3 Sigma" rule (*meshulashin?*) says that almost all data from a Normal Distribution (bell-shaped curve) lies within 3 Standard Deviations (i.e. 3σ) of the Mean.²⁷ Thus σ is roughly estimated for a distribution by dividing its range by 6. As a result, for the *Baraisa's* case of 80, 100 and 120 the "3 Sigma" rule yields a σ estimate of $6\frac{2}{3}$ $((120-80)/6)$ and the choice of $93\frac{1}{3}$ is tantamount to choosing as the assessment a value that is exactly one standard deviation below the mean value²⁸ of 100.

Our explanation of what Acherim hold in the symmetrical case preserves the *Gemara's* basic contention that Acherim place the assessment at $93\frac{1}{3}$, albeit not for the reason the *Gemara* gives. In this way the *Gemara* may have had a Mesorah on the amount Acherim said, but not on the reason. Thus, when Rav Ashi says he cannot understand Acherim's reason, he in effect is saying that he cannot halachically follow the Mesorah without understanding the reason. By us explaining the rationale of Acherim in a more intuitively (statistically) appealing manner it justifies Acherim's position and allows it to be halachically applied.

²⁷ This rule is generally extended to include non-Normal symmetrical unimodal-distributions. For a Normal distribution about 99.7% of the population lies within three standard deviations of the Mean. For "all" distributions there is the more general Chebyshev Rule which says that at least 89% of the data lie within 3 standard deviations of the Mean.

²⁸ While the "middle" concept of Mean and Median dates back to the Greeks and presumably might have been known to the Chachamim, the reasoning they used to arrive at their conclusions seems to be driven by a different set of logic (majority rule). The standard deviation is of much more recent vintage and was certainly not formally known to the Chachmei HaTalmud. Chebyshev Rule (see previous footnote), tying the percent of the numbers in the list to the number of standard deviations they are from the center, was not formulated until 1867. Nevertheless, we have shown that by using an alternate analysis the Chachmei HaTalmud could have come up with a solution that parallels some elements of this later mathematical discovery.

A Final Thought

In this final section of the paper we would like to discuss a broader implication of the way we have explained the *Gemara* in *Bava Basra*. Our explanation has the Tanna Acherim promulgating a position far more complex than the one the *Gemara* attributes to them. While there is much leeway in explaining a *pasuk* in Chumash differently from the way the *Gemara* does,²⁹ as well as explaining a Mishna far differently from the way the *Gemara* does,³⁰ the caveat is always that the difference in explanation may not change a halacha. In this case our explanation of Acherim, particularly in the asymmetrical case, results in a different numerical valuation of a property from the one the *Gemara* offers. Indeed, had the *Gemara* not concluded with its statement that Rav Ashi “did not understand Acherim,” we would not have offered our explanation no matter how many questions the *Gemara*’s explanation of the *Baraisa* raised. However, in this case Rav Ashi’s concluding remark rejects the *pesak* previously given by the *Gemara*. Moreover, neither Rav Ashi nor the *Gemara* offers an alternate opinion as to what to do and in fact Rambam does not conclude according to Acherim. In such a situation we feel there is no problem with our offering a different explanation.³¹ Since the *Gemara* does not end with a halachic decision, our explanation does not contradict the *Gemara*’s conclusion. ❧

²⁹ See, e.g., *Ohr HaChaim Devarim* 32:1.

³⁰ See, e.g., Rambam *Perush HaMishnah*, *Nazir* 5:5.

³¹ In many ways what we are doing is analogous to an explanation of the Mishna *Kesubos* 93a given by Nobel Prize winner Prof. Yisrael Aumann “Game Theoretic Analysis of a Bankruptcy Problem From the Talmud,” *Journal of Economic Theory*, Vol. 36, Issue 2, August 1985, pp. 195-213. The Mishna there describes 3 scenarios of how much money to pay 3 debtors of different amounts, from an estate that cannot meet the full debt. The *Gemara* challenges the Mishna’s recommendations and concludes with Rebbe ostensibly rejecting the Mishna’s conclusions. The Rishonim discuss many difficulties with the *Gemara*’s presentation and are unable to offer a satisfactory explanation of the Mishna’s recommendations. Prof. Aumann showed that using modern Game Theory the Mishna’s recommendations are perfectly consistent.

Appendix

Bava Basra 107a

תנו רבנן שלשה שירדו לשום אחד אומר במנה ושנים אומרים במאתים אחד אומר במאתים ושנים אומרים במנה בטל יחיד במיעוטו אחד אומר במנה ואחד אומר בעשרים ואחד אומר בשלשים נדון במנה רבי אליעזר ברבי צדוק אומר נדון בתשעים אחרים אומרים עושין שומא ביניהן ומשלשין מאן דאמר נדון במנה מילתא מציעתא רבי אליעזר בר' צדוק אומר נדון בתשעים קא סבר הא ארעא תשעין שוה והאי דקאמר עשרים דקא טעי עשרה לאחוריה והאי דקא אמר מנה קא טעי עשרה לקמיה אדרבה האי ארעא מאה ועשרה שויה והאי דקאמר מנה קא טעי עשרה לאחוריה והאי דקאמר שלשים קא טעי עשרה לקמיה נקוט מיהת תרי קמאי בידך דמתורת מנה לא מפקי ליה אחרים אומרים עושין שומא ביניהן ומשלשין קא סברי האי ארעא תשעין ותלתא ותלתא שויה האי דקא אמר עשרים קא טעי תליסר ותלתא לאחוריה והאי דקאמר מנה קא טעי תליסר ותלתא לקמיה ובדין הוא דלימא טפי והאי דלא קאמר סבר מיסתאי דקא מטפינא כולי האי אחבראי אדרבה הא ארעא מאה ותליסר ותלתא שויה האי דקאמר מנה קא טעי תליסר ותלתא לאחוריה והאי דקאמר שלשים טעי תליסר ותלתא לקמיה ובדין הוא דקאמר טפי סבר מיסתאי דקא מטפינא כולי האי אחבראי נקוט מיהת תרי קמאי בידך דמתורת מאה לא מפקי ליה אמר רב הונא הלכה כאחרים אמר רב אשי טעמא דאחרים לא ידעינן הלכתא עבדינן כוותיהו תנו דייני גולה עושין שומא ביניהן ומשלשין אמר רב הונא הלכתא כדייני גולה אמר רב אשי טעמא דדייני גולה לא ידעינן הלכתא עבדינן כוותיהו:

English Translation

Our Rabbis taught: three went down to assess, one says it is a *maneh* {100 zuz} and two say 200 {zuz}, one says 200 and two say a *maneh*, the one being in the minority is overruled. One says a *maneh*, one says twenty {80 zuz}, and one says thirty {120 zuz}, it is adjudged at a *maneh*. R. Eliezer b.R. Zadok, said: It is adjudged at 90. Acherim said: we make an estimate between them and are *me-shalschin* {divide into three parts}. He who said, 'It is adjudged at a *maneh*,' adopts the middle course. R. Eliezer b. R. Zadok, who said, 'It is adjudged at ninety,' opines that the land is worth 90, and he who said twenty made a mistake by 10, and he who said *maneh* made a mistake by 10. On the contrary! Assume it is worth 110 and he who said a *maneh* underestimated it by 10 and he who said thirty overestimated it by 10? Adopt the first two lower values, since nei-

ther exceeds the sum of one *maneh*. Acherim said to make an estimate and be *meshalschim* holds the land is worth $93\frac{1}{3}$; he who valued it at twenty underestimated it by $13\frac{1}{3}$; he who valued it at a *maneh* overestimated it by $13\frac{1}{3}$. Logically the latter should have given a higher estimate but did not do it because he thought, 'It is enough that I have exceeded my colleague's by so much.' On the contrary! Assume the land is worth $113\frac{1}{3}$; he who valued it at a *maneh* underestimated it by $13\frac{1}{3}$, and he who valued it at thirty overestimated it by $13\frac{1}{3}$; logically he should have submitted a higher estimate. He thinks, 'It is enough that I have exceeded my colleague's by so much.' At all events one should adopt the first two, since both do not exceed a *maneh*. R. Huna said: The *halachah* is like Acherim. R. Ashi said: We do not know the reason of Acherim; shall we administer the law in accordance with their view? The judges of the Exile taught: The difference between them is calculated and divided by three. R. Huna said: The law is in accordance with the Judges of the Exile. R. Ashi said: We do not know the reason for the opinion of the judges of the Exile, shall we administer the law in accordance with their view?