A. A. Fraenkel's Philosophy of Religion: A Translation of "Beliefs and Opinions in Light of the Natural Sciences" [1]

By: M. ZELCER

Introductory Essay

1 – Life

Abraham Adolf Halevi Fraenkel is best known to mathematicians and philosophers as one of the founders of modern set theory. From the late 1800s through the 1930s, modern logic and set theory emerged as part of the new program to establish reliable and secure foundations for mathematics. Logicians and set theorists were then devising the methodology that would shape the way mathematics is currently practiced. Mathematicians and philosophers like Georg Cantor, Gottlob Frege, David Hilbert, Bertrand Russell, Alfred N. Whitehead, Richard Dedekind, Ernst Zermelo, and Kurt Gödel were making fundamental contributions to the foundations of mathematics. Though his early mathematical work was in the field of algebra, Fraenkel's most notable contribution was in the theory of sets. He and Ernst Zermelo formulated a set theory that should be not susceptible to the famous paradox of Russell, or the Burali-Forti

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1 המאמר הזה הוא לז"נ סבי, ר' ירחמיאל זעלצער ז"ל.
Thanks to Shaul Katz for bringing Fraenkel's article to my attention many years ago in a discussion about the history of the Hebrew University; Heshey Zelcer for his assistance with the translation; Dahlia Kozlowsky for stylistic comments; and Noson Yanofsky for many valuable suggestions; a few footnotes are due entirely to him. Also, thanks to Tina Weiss at the HUC library for help tracking down some of Fraenkel's essays.

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paradox.[2] The set theory he helped develop is the most popular and is known as Zermelo-Fraenkel set theory, or ZFC. To understand the relevance of ZFC consider that the vast majority of modern mathematics can be formulated with, and be seen as being built upon, sets and ZFC's few simple axioms is their foundation. Most working mathematicians usually do not think about the axioms, nor do they care if their work can be put into the language of ZFC. Nevertheless, with enough effort, their work can be stated within the language of ZFC. From this important position, the axioms of ZFC can be seen as the axioms of all of mathematics and hence the axioms of exact reasoning itself.[3]

Fraenkel was born in Munich in 1891 to a fairly well known Orthodox Jewish German family whose lineage includes people such as his great-grandfather B. H. Auerbach, the publisher of the (now infamous) *Sefer ha-Eshkol*.[4] Like many German students, he studied in various universities, including the universities of Berlin, Munich, Marburg, and Breslau before receiving his PhD in mathematics. He served as a German soldier, mostly in a medical capacity, for 50 months in WWI. During that period he met many kinds of Jews and developed the Jewish world-view that would stick with him for the rest of his life. Shortly after returning from war, he met Wilhelmina Malka A. Prins who was studying German at the time. They married in 1920. He thought their partnership was ideal be-

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2 The paradoxes that show that Cantor's original set theory is inconsistent. Russell's paradox from 1902 (first discovered by Zermelo) is similar to the "Barber Paradox." If there is a lone barber in an isolated town who shaves all and only those people who do not shave themselves, who shaves the barber? If he shaves himself, then he doesn't shave himself, and if he doesn't, then he does. In set theory, you can have a "barber set" that is made up of those kinds of elements that cause analogous problems. The Burali-Forti paradox (1897) is more complicated and involves the largest ordinal number in the set of ordinal numbers.

3 The discussion of ZFC is loosely adapted from Noson Yanofsky's forthcoming *The Outer Limits of Reason*.

4 (Fraenkel 1967, 13) fails to mention the controversy over this work. It is possible Fraenkel was not aware that it might have been a forgery. See Mark B. Shapiro's note for more on the provenance of *Sefer ha-Eshkol*: <http://seforim.traditiononline.org/index.cfm/Besamim%20Rosh>.

cause apart from religion and Zionism, they had no interests in common. They complemented each other perfectly and would eventually have four children together. After the war, Fraenkel taught mathematics in the university of Marburg from 1922 and then in Kiel from 1928. Though he claims to have experienced no anti-Semitism at either Marburg or Kiel prior to 1933 (Fraenkel 1967, 185) he decided to try teaching in the newly opened Hebrew University in Jerusalem. He stayed there for two years from 1929 to 1931, but was compelled to return to Kiel because of the poor economic situation in Palestine. When the Nazis came to power a year later he was forced to leave Kiel. The American Friends of Hebrew University sponsored his professorship and he was able to return to Hebrew University where he remained until his retirement in 1959.[5] In mandate Palestine and then in Israel, he was a member of various national boards and councils of the Yishuv (Jewish Settlement in Palestine) and was an indefatigable educator throughout.

Fraenkel was initially hesitant to join the newly opened Hebrew University as it was Jewish but avowedly secular. He recounts in his autobiography that he wrote to Rabbi Abraham Isaac Kook—a man for whom he expresses great respect (see Fraenkel 1967, 191)—asking if it was appropriate, given its secular Jewish orientation, to join the faculty of the Hebrew University. Fraenkel was worried that the Hebrew University would be a forum for heretical "scientific" studies of the Bible and Jewish sacred texts.[6] Rav Kook

5 Two useful books about Jewish mathematicians and mathematics under the Nazis are (Segal 2003) and (Siegmund-Schultze 2009). The latter book also contains references to archives that contain material and biographical information about Fraenkel, as does (Katz 2004) and (Katz 1997).

6 (Fraenkel 1918) first addresses this question. (The article was published with a response by Harry Torczyner, who coincidentally would later go on to win the Israel Prize in the same year as Fraenkel.) A few years later he raises related questions in (Fraenkel 1924). That article generated a number of replies in the following issue of the journal, some (e.g., that of Ch. Tschernowitz) quite sharp, to which he responded a few months later in (Fraenkel 1924b). (Fraenkel 1924) also reproduced a letter from Rabbi D. Hoffmann, the head of the Berlin Rabbinical Seminary, to Fraenkel, in support of a Jewish university urging the Orthodox to participate in such a university so as not to be absent from any debate (cf. Rav Kook's

characteristically replied that it was indeed Fraenkel's responsibility to participate in the university and thereby elevate its spiritual level. Asking R. Kook about joining the faculty of a Jewish and secular Hebrew University was no case of false piety. Fraenkel was deeply bothered by the nature of the university years before it opened its doors and wrote a number of articles grappling with this dilemma. In 1924, prior to the opening of the university, he argued that it would be offensive to the believing Jew to establish an institution in the heart of the Jewish homeland that took stances on religious topics like Bible study that were staunchly antithetical to the religious viewpoint: "…teachings dissenting from Judaism have cheerfully been given hospitality in this or that Seminary or Yeshibah (sic), but the adoption of a definite viewpoint in *the locality which aims to embody the highest scientific instance of universal Judaism*, will rightfully be considered by the Jew everywhere throughout the world who is convinced of the opposite, and doubly so by the Palestinian Jew, as the gravest insult to his holiest feelings, and will be correspondingly combated. To conjure up such a cultural struggle means to assume a terrible responsibility" (Fraenkel 1924: 30, *emphasis* in original).

In addition to R. Kook, Fraenkel's autobiography recounts contact with many interesting Jewish, Zionist, and scientific figures.[7] Not only was he acquainted with many of the more interesting personalities, he was also engaged with them on a religious level. He was called upon to weigh in on one of the more famous Jewish legal questions of the day: what day to fast on Yom Kippur in Japan? This question was hotly debated when in 1941 Jewish refugees who found themselves in Japan during World War II sent an urgent telegram to Rabbi Herzog, the Chief Rabbi of Palestine, to verify the correct day to fast for Yom Kippur. The question hinges on wheth-

reply below). Hoffman also felt it was important to avoid a "culture war" between the religious and secular.

[7] For example, he corresponded with Niels and Harald Bohr, Husserl, Koyré, von Neumann, and especially Einstein, who eventually convinced Fraenkel to consider alternate views in the philosophy of mathematics. Fraenkel also mentions the many talks he had with Einstein about religion (Fraenkel 1967, 172).

er the International Date Line tracks the change from one day to the next in the same place as Jewish law. An immediate meeting was called with some of the most prominent rabbis of the time. Fraenkel was asked to participate most likely because of his combined knowledge of Jewish Law and the mathematical issues of calendar systems.[8] He sided with the majority, against the Ḥazon Ish, that the local days in Japan are to be treated as those same days for religious purposes.[9]

Fraenkel had a strong desire to see the Jews united in Israel. In a speech he gave when accepting an honorary doctorate from Dropsie College in 1946 (Fraenkel 1946), he explained how one can fully appreciate many passages in the Bible, Talmud and Midrash only by being present in the Holy Land. He gives a number of examples. The first he cites is a verse from *Song of Songs* (1:14): "My beloved is unto me as a cluster of henna in the vineyards of Ein Gedi." After going through a vivid description of what it was like to travel to Ein Gedi in the 1940s, he tells us that when one reaches this desolate place there is one area in which he will find clusters of tropical plants, including the henna tree. He then explains that the simple meaning of the verse is that the bride likes to contrast her loved one with the other men in the country, comparing him to the henna tree and everyone else to the dry and salty plants of the surrounding desert. In another example, he offers up a scientific fact about Israel to explain away an apparent midrashic exaggeration on Exodus (9:33) about the miracle of the plague of Hail in Egypt. In Exodus it states that when the plague was over "Moses went out from the city, from Pharaoh, and spread forth his hands unto the Lord, and the thunders and hail ceased and the rain was not poured to the earth." The Midrash asks why is the final word "ארצה", "upon the earth," necessary? The verse could have just read "the rain poured

8 Fraenkel wrote a number of works on religious calendar systems including: (Fraenkel 1908), (Fraenkel 1909), (Fraenkel 1910), (Fraenkel 1945), (Fraenkel 1958), (Fraenkel 1966), and (Fraenkel 1969). His interest in calendars was likely related to his research on the mathematician Carl Friedrich Gauss who was interested in this as well.

9 The entire issue is recounted in (Kasher 1976/7, see esp. page 247). See also (Fraenkel 1941).

down no more." The Midrash then answers that even the rain that was in the air, coming down, did not reach the ground. But, Fraenkel asks, do the Rabbis really need to enhance the miracle in this way? To which he answers that the Rabbis are not exaggerating. He describes standing on the Mount of Olives on a rainy day, looking down upon the Jordan valley, experiencing the meteorological phenomenon of "adiabatic heating." There is little rainfall in the Jordan valley, a mere 15 miles from Jerusalem, because the rain will often evaporate before it hits the ground. This happens because the rain has a much further distance to travel in the valley and in the process gets heated to the point of evaporating. The Rabbis thus saw this as a natural explanation for a certain meteorological phenomenon, not a new miracle. Fraenkel concluded the speech with a plea for students to come spend time and experience the Holy Land for themselves.

In 1956 Fraenkel was awarded the Israel Prize, the nation's highest honor, for his work in the exact sciences. His influence on the culture of Hebrew University and mathematics in Israel[10] is still felt today, especially in the attitude toward pure mathematics[11] and advanced studies in mathematics. Some of Fraenkel's students at Hebrew University also became extremely important mathematicians. Perhaps the most famous of these was Abraham Robinson, the creator of nonstandard analysis.

Abraham Fraenkel died unexpectedly on October 15, 1965; he worked on mathematics and Jewish education to his last days.

2 – Jewish Interests

Fraenkel's mathematical papers and books are well known.[12] His Jewish writings and his expository philosophical papers are consi-

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[10] See (Fraenkel 1947) for Fraenkel's account of the relationship between Hebrew University and secondary education in the Yishuv, especially during his tenure as chairman of the Secondary Schools Committee.

[11] Fraenkel's influence on the mathematics of Hebrew University and other details of his life are discussed in (Katz 2004). See also (Dauben 1995).

[12] A list of Fraenkel's most important mathematical papers can be found in the festschrift for him (Bar-Hillel, *et al* 1961, ix–x) and in the bibliography of (Fraenkel 1953b).

derably more obscure. He wrote a number of articles of Jewish interest[13] and an autobiography (Fraenkel 1967), only part of which has been published posthumously, and only in German. Since it contains anecdotes and information about many of the prominent mathematicians that Fraenkel interacted with, it is often used by scholars of German pre-war mathematics. It also contains much information about Jews in the Weimar Republic. Perhaps the most famous anecdote in the book recounts an exchange between David Hilbert, arguably the most important mathematician of his era, and the over-zealous Nazi science and education minister Bernhard Rust. Rust had asked Hilbert if it was true that German mathematics suffered when Jews and their sympathizers were removed from his university. To which Hilbert replied, "Suffered? It has not suffered, Mr. Secretary; it does not exist anymore!" (Fraenkel 1967, 159).

Fraenkel was also involved, as both an editor and contributor, with the now defunct mathematics journal *Scripta Mathematica*, edited out of Yeshiva University. The journal was unique for its day in being edited by professional mathematicians but geared toward educated lay-readers. Some of the articles were of Jewish interest, though its primary ambition was to be a forum for writing about the history and philosophy of mathematics. Fraenkel contributed a dozen articles, most of them on philosophical-foundational

[13] Some of Fraenkel's writings that are of Jewish interest that are not mentioned otherwise in this essay include a brief forward to (Feldman 1931), a 46-page pamphlet (with a brief appendix by Rabbi Herzog) for popular audiences on the problem of the birthrate in the Yishuv and the impact on Jewish demographics (Fraenkel 1943), and (Fraenkel 1955b) which deals with the implications of the State of Israel on the prayer services. Fraenkel also mentions (Fraenkel 1967, 133) other articles he wrote while serving in WWI: "Jüdische Eindrücke im Feld" ("Jewish impressions from the field"), "Gedanken über künftige Entwicklung und Aufgaben der Agudas Jisroel" ("Thoughts on future developments and tasks of the Agudas Yisroel"), and "Thoratreues Judentum und Zionistische Organisation" ("Torah-True Judaism and Zionist organization"). The last article resulted in a heated polemic with the anti-Zionist editor of the *Agudah-Blätter* and his break with Agudah.

areas of mathematics.[14] One of the articles, (Fraenkel 1960), is "historical," containing brief biographical sketches of many prominent Jewish mathematicians of the Nineteenth Century. He also penned in *Scripta Mathematica* an obituary of his uncle (by marriage), the mathematician Alfred Loewy (Fraenkel 1938).[15] Fraenkel was also the mathematics editor for the *Encyclopedia Hebraica* and contributed a number of articles including one on the calendar (Fraenkel 1969) and another on geometry.

Fraenkel complained that when he gave his first talk on mathematics in Hebrew University there was not enough mathematical terminology developed to lecture in Hebrew. His mathematics textbook (Fraenkel 1953) helped establish the mathematical vocabulary still in use today. He was known as a strong advocate of the Hebrew language, and for exhibiting disdain for idiomatic Hebrew. One story has him complaining to a bus driver who told his passengers "להתקדם אחורה" ("to go back," lit., "to go forward to the rear").

3 – The Essay

Fraenkel wrote "Beliefs and Opinions" around 1930, revisiting some ideas from an article he wrote some 12 years earlier. This was a very exciting period for mathematics and science. Logic and its role in the foundations of mathematics were still new and he was an important member of the generation of mathematicians fashioning them. Physics and cosmology were undergoing revolutions. Einstein's papers on relativity had been around for some time, but they were still somewhat controversial, especially in Germany. New discoveries were just being made in many areas of chemistry, particle physics, and astrophysics; and the field of quantum mechanics was

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14 A few of the articles translate chapters of his influential (Fraenkel 1953), which is still used today in Israel. Some of these articles were also collected as (Fraenkel 1955).

15 Fraenkel wrote a number of obituaries in Hebrew for his friends and colleagues including one for the Protestant theologian Rudolf Otto (*Haaretz* 6 March 1937) and another for Edmund Landau (*Haaretz* 4 March 1938), one of the most prominent mathematicians of his generation and likely the great-grandson of the Noda be-Yehuda (a strong advocate of mathematics as part of Jewish learning) or of his brother.

just being developed. Fraenkel, picking up a point that Thomas Kuhn would exploit thirty years later, noted that science evolves. The idea that science grows is not new, but Fraenkel stressed that the scientific understanding of the fundamental laws of nature are periodically overturned. (He is returning here to an idea he broached in his (Fraenkel 1918b).) Kuhn would later describe this phenomenon more fully in a process he called "paradigm shifts" in the history of science. Kuhn (Kuhn 1962) famously argued that science undergoes periods where all respectable scientists in a field are actively engaged in one research program and there is broad agreement about what scientific objects exist, which laws govern them, and what methodology ought to be used to study them. Then, along comes a Newton, an Einstein, a Dalton or a Darwin, and radically changes the way that scientists see their field. These revolutionary scientists do not merely fill in details that were not yet known. Rather, they give us a new way of looking at a field that forces us to throw out the old text books and re-conceive the nature of the discipline with new terminology, methodology, ontology, concepts, and laws. The older scientists are unable to communicate with the younger ones about the discipline because they are no longer talking about the same things. And that, Kuhn tells us, is how science "progresses."

There is a sense in which Fraenkel himself was contributing to something like a revolutionary period in mathematics, and he was well aware of it. Fraenkel exploits the evolutionary features of the history of science to explain why science should not be given the last word, especially when it contrasts with Torah. Fraenkel talks about "pictures" (תמונות) of science,[16] which bear some similarity to Kuhnian paradigms. A "picture" for Fraenkel is a description of the state of a science. It is a unified view of how a set of scientific phenomena *behave*; it is not a mechanistic explanation of the nature and essence of the phenomena. Scientists compose these pictures but discard them from time to time and put together new ones. Even central concepts like causality and continuity, which seem to be

[16] For consistency, I render "תמונות" as "pictures" consistently throughout the translation, even at the occasional expense of some awkwardness where "model" or "image" may be more appropriate.

part of what we just assume about the scientific method, are part of a "picture" which in many cases have been discarded or changed. For Fraenkel, the Torah's truths are eternal, and cannot change. Since the Torah has to give us an unchanging picture while the scientific picture changes, it makes little sense for the Torah to provide scientific facts. If it did, people would have to change their views of Torah each time science changed. So the Torah speaks in the language of man and leaves it understood that interpretation may be necessary. Many of Kuhn's followers took the lesson of paradigm shifts in science to be that there is no scientific truth. After all, if science constantly changes, how can it lay claim to eternal truth? Fraenkel, however, makes a weaker claim in this essay. Fraenkel does not claim that there is no truth, rather he claims that scientists are constantly facing problems finding out what it is. The Torah therefore can be seen as providing us with some truth that science may eventually catch up with when it hits on the correct picture. In some sense, Fraenkel's argument in the essay for the universe's finitude is a clear case of this.

But even if science does discover true laws of nature, we are still not guaranteed to have the complete story. It is possible to have scientific laws that seem to work, yet also have other forces at work at the same time that are not accessible to science. Fraenkel uses the notion of "layers of laws." Nowadays philosophers often speak about one set of laws "supervening" over another. Here is an example: When we study the human body, we assume that there are a set of laws of nature that determine how the body will react to different stimuli, like a certain drug. But we also know that the human body is made of molecules and atoms that follow the laws of physics like all other particles in the universe. So we can have a set of laws of biology "layered upon" a set of laws of physics: biology "supervenes" over physics. They have different sets of laws that are completely compatible with each other, yet they each have their own explanatory schema and "picture." The laws of physics are rarely invoked to explain biological facts, and of course, vice-versa. But both are scientifically useful. More interestingly, there is some sense in which the laws governing molecular behavior are (or at least were before quantum mechanics was taken into account) often taken to be deterministic, meaning that there was a precise reaction to each action and nature does not deviate from this. However, the laws governing biology are often taken to be probabilistic. (E.g., we can accept the fact that there is a law of nature that dictates that a drug can work for only a given percentage of the cases.) So we have a set of probabilistic biological laws governing a body that is made up of particles that seem to be governed by deterministic laws that in turn supervene over quantum mechanical entities that are probabilistic individually, but deterministic when viewed on a larger scale. Fraenkel stresses that this is what is happening in nature. There are the layers of physical law that science countenances and then there may also be a layer of laws that science does not have access to that also influences the world. In any case, the fact that current science does not support a certain view should not be taken to mean that it is incorrect or impossible.

Reconciling science and religion has been a central part of Jewish philosophy at least since Philo of Alexandria. It was central to medieval philosophy as well. This is not the place to evaluate the various approaches that attempt to reconcile science and Torah or to taxonomize the responses and situate Fraenkel's approach.[17] Contemporary discussions within Orthodox Jewish literature span the gamut from less to more sophisticated and include a variety of approaches. Some reinterpret the Torah metaphorically, some reinterpret scientific concepts, like time, to fit the meaning of the Torah, others look at the value of science more than the proofs it proffers, and still others insist that Torah and science make up separate domains that ought not to be compared, let alone reconciled. There are elements of some of these approaches in Fraenkel's article. Because the article is now about 80 years old, a long time given the pace of modern science, there was much Fraenkel could not have anticipated. But his core ideas are still interesting, and he may have been the first to express some of them. It certainly confronts a recent complaint by an important Jewish philosopher, that those who do Jewish philosophy in the modern period do not understand the relevant logic and science (Samuelson 2009). These are issues that Fraenkel thought about deeply and for a long time, and to which he

[17] (Rosenberg 1987–8) does this to some extent. The interested reader is directed there.

brought immense mathematical sophistication. If nothing else, we hope that bringing this translation to a wider audience in English will shed light on the religious ideas of a first-rate mathematical thinker and acknowledge Fraenkel as someone whose life was dedicated not only to mathematics, but to Jewish thought and the intersection of the two.

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The article below was first published in Hebrew as (Fraenkel 1930-1) and is translated here for the first time.

Beliefs and Opinions in Light of the Natural Sciences[18],[19]

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I

The purpose of the natural sciences is not to *explain* the essence of phenomena found in nature, but rather to *describe* them with a unified theory, and to include phenomena which appear disparate and unique under more general phenomena.[20] In this vein the phenomenon of the rainbow is based on the refraction of light which in

18 All footnotes are the translator's except where explicitly stated. Some of Fraenkel's notes have been incorporated into the text.

19 The title may have been adapted from a chapter heading in (Kook 1920, 129) "למלחמת הדעות והאמונות". The present work seems to be Fraenkel's scientific take on a theme elaborated on there.

20 Questions about the nature of science, including what is a scientific explanation, what is a scientific theory, and what is the general methodology of science, became central for the Positivist school of philosophy which flourished in Austria and Germany between the world wars and then in the US and England for a while after WWII. Here Fraenkel is alluding to a view—that unification of diverse phenomena is the goal of scientific explanation—that would become associated with the view in (Friedman 1974). Fraenkel's skepticism about what scientific explanations can provide is already found in his (Fraenkel 1918b).

turn is based on the speed of light in different conditions (or mediums). Or consider another example: the phenomenon of a stone falling toward the Earth, and the phenomenon of the revolution of the moon around the Earth are both explained by the same principle or phenomenon: the Earth's gravity. The exact natural sciences (e.g., physics and chemistry) are superior to descriptive sciences (e.g., biology) specifically with respect to quantitative investigations—which are present in the former but are lacking in the latter. It is therefore self-evident that scientific theories, even in physics, do not provide us with actual descriptions of phenomena as they really are, but rather only *pictures* [תמונות] of them. The value of any scientific theory is therefore dependent on the quality of the picture [תמונה] it provides. It is therefore not unusual or surprising when different sciences (say mechanics and the theory of electricity) provide pictures that contradict each other. Scientists of course sense the contradictions in these pictures but they hope that eventually better pictures will emerge at which point they will replace today's pictures with newer ones, and the contradictions will be resolved. But for the moment they are forced to use the pictures specific to each science. (*A fortiori* [ק"ו], it should not be surprising if, at a certain period, there are contradictions between scientific pictures and the eternal truth of our Torah.)

As a rule, scientific theories are merely pictures of phenomena, not truths. Also, views that once seemed to be eternal, self-evident and not in need of any proof, have been suddenly refuted and discredited; they ceded their position to better, more accurate pictures. This is how the theory of relativity overturned the idea of simultaneity which no one had ever doubted.[21] According to today's picture, it is as if each part of space carries its own time [frame], and we cannot compare time frames of different moving objects. Chemists, who classified all matter into absolute separate elements, mocked the alchemists who had claimed the ability to turn certain sub-

[21] Whereas two observers can agree on a time for a single event that they both witnessed, one of the significant repercussions of the special theory of relativity was to show that it is impossible for separate observers to say in an absolute sense whether or not events that took place in different places happened at the same time. Cf. (Fraenkel 1918b).

stances into gold.[22] Then the discovery of radioactive materials did away with the concept of permanent chemical elements. The prevailing picture today is that, in principle, all elements are compounds of one substance (the word "substance" here is imprecise, but this is not the place to elaborate).[23] Furthermore: quantum theory removed the concept of continuity from nature. Previously the prevailing rule was that "nature does not jump;" today's best picture is that "nature jumps." Known changes in atomic energy levels are most reasonably explained by assuming that there are tiny leaps that are added together.[24] Lately, we have been forced, even in principle, to give up the law of causation[25]—a principle that has always been solid. It turns out that in principle we cannot precisely investigate the parts of the tiniest material (parts of the molecule or atom). To explain the tiny changes, we assume that (non-causal) spontaneous processes which subsequently join together as larger processes appear as causal changes. [26]

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22 Cf. (Fraenkel 1918b).

23 Since Fraenkel wrote this, chemistry has gone through a number of significant developments and changes, though it is still believed that all of nature is built of one kind of "substance." It is still an open question, however, what that substance is. Two candidates are quarks and leptons, and strings though the theories behind these were developed after Fraenkel wrote this essay.

24 The prevailing model in physics is that an atom is a bundle of protons and (usually) neutrons with "shells" of electrons orbiting them. Each shell holds a certain number of electrons that, under certain conditions, move from one shell to the next. When an electron moves from one shell to the next, it is never in between shells. The electron thus "jumps" from one shell to another without ever existing in between the two shells.

25 In quantum mechanics the laws are given probabilistically as opposed to deterministically. That is, the laws of quantum mechanics for systems of subatomic particles give probabilities for the ways that the system will evolve. There is no way to tell exactly how the system will evolve. This randomness destroys the notion of causation that had been so central to science. It is important to keep in mind that since objects in the universe are made up of large ensembles of subatomic particles, this randomness will not be apparent.

26 Keeping in mind the previous footnote: This phenomenon is similar to the fact that if a fair coin is tossed 1000 times, we will not be able to determine what the outcome will be for each toss, but we can say with con-

On the other hand, when the Torah describes the formation of the world, and more generally when it speaks about what goes on in nature, in principle, it uses the rule that "the Torah speaks in the language of people."[27] It is not only that this rule is suitable with respect to the purpose of the Torah, which is not to explain the natural sciences; rather it is essential that it be this way. Had the Torah described the precise processes of natural science, then it would not have been understood in the periods prior to the discovery of that level of science. Each generation would have had to change its stance vis-à-vis the Torah in accordance with the progress of their scientific theories.

There are various ways, from a scientific point of view, to understand the age of the Earth and the process of Creation as it is narrated in the Torah. We will list several of these without preferring one over another. The accepted picture in geology and other natural sciences today is that many eras had to pass for the Earth to evolve to its present shape. This hypothesis fits quite well with a saying of Rabbi Abahu: "God created worlds and destroyed them before creating this one."[28] Already in Talmudic times, there was a need to understand creation in the book of Genesis not literally, and not "in the language of people,"[29] but rather that there was a phenomenon of creating and destroying worlds before our world was finally created.[30]

siderable definiteness that the coin will land on heads about five hundred times. So cumulatively, nature appears causally determinate.

27 דברה תורה כלשון בני אדם (the Torah speaks in the language of people) is a common rabbinic expression sometimes used to explain why a Biblical text should not be used hermeneutically to justify more than its apparent meaning. See, e.g., B. Talmud *Niddah* 44a, *Ketubbot* 67b, and *Avodah Zarah* 27a. See also Maimonides' *Guide to the Perplexed* I, 26 and *Yad*, *Yesodei HaTorah* 1:12.

28 *Genesis Rabbah* 3:7.

29 It is worth noting that in *The Guide to the Perplexed* (II, 25) Maimonides argues that existing proofs for the eternity of the world are flawed. Nonetheless, should it be proven that the world is eternal, contrary to the literal reading of the text of the Torah, he would interpret the Torah figuratively.

30 It should be noted that it is only relatively recently that it has become widely accepted that the universe is of a finite age. (See below.)

Indeed, on examination, it is clear that it is simply impossible to interpret the words "one day", "second day"[31] at face value, as a period encompassing 24 hours from one day to the next,[32] since there is no fixed time scale that can be used. Measurements of time are either subjective, in which case they negate any objective basis, or they are based on physical-astronomical phenomena, i.e., on the rotation of the Earth and its movement around the Sun. Yet the Torah tells us[33] that the procession of the zodiac—"the luminaries"—were fixed and arranged only on the fourth day. It therefore follows that it is meaningless to ask whether a "day" in the first days of Creation, before the heavenly bodies were established, correspond to our day or a period of millions of years. This is because both possibilities lack any value, since man is unable to grasp or establish the concept of time so long as the motion of the heavenly bodies has not been fixed.

If this explanation fits the picture accepted by the natural sciences today and it accurately explains Creation as described in the Torah, there is still another important point, one that undermines the core geological assumption. Keep in mind the dilemma of continuity in nature. It is possible to draw a conclusion about the trajectory of a whole line based on a partial path (e.g., a section of an arc) only if it is clear that no part of the line deviates [from the arc]. So too, our precise observations of nature and our experiments and calculations are based on the fundamental assumption that nature is continuous and uninterrupted.[34] This is an iron-clad assumption for all who investigate nature. We know, however, that conventional chemical assumptions regarding chemistry and physics do not apply under conditions of elevated pressure and temperature, because extreme pressure and heat interrupt the continuous progression of natural processes. According to both the Biblical account and the scientific cosmogonical[35] theories, the material of the Earth had to be in such condition at the beginning of Creation.

31 From the verses in Genesis 1:5 and 1:8 respectively.
32 Cf. Psalms 90:4.
33 Genesis 1:14–19.
34 Cf. (Schneerson 1961).
35 I.e., theories of the genesis, or origin, of the cosmos.

Thus, for example, even at relatively moderate temperatures, an immediate bond is formed between the solid and gaseous states of matter. This explains why we may view the interior of the Earth as solid, molten, or gaseous. It is plausible that extreme pressure and heat accomplishes in a short period of time (even in a moment), that which would take millions of years under today's conditions of pressure and heat. Furthermore, all our calculations and theories about "what happened since the beginning" are only valid as far as the point in time from which there was uninterrupted continuity of the events. Prior to this point continuity was broken and so too our calculations. Similarly, the theory of evolution, for example, can only claim that from the start of continuity, organisms appear to have evolved from a single unified source.

II

Part I ended with a critique of the geological assumption that it took millions of years for the Earth to form. We argued that all our calculations and theories are valid only from such point in time that nature proceeded continuously without any [cataclysmic] interruption. Prior to that point in time, given the conditions of extreme pressure and heat, it is plausible that changes that require millions of years under our conditions of temperature and heat, happened within a very short time, perhaps even in a few moments. This conclusion contradicts the geological assumption of long, almost unlimited, geological eras, and replaces it with an assumption of the creation of the Earth in a relatively short period of time. Should materialist scientists quietly suggest their theory about the eternity of the world based on an acknowledged existence of matter and energy, reply to them that this theory is also contradicted by the laws of thermodynamics; but admittedly this requires some introduction.[36]

[36] Part I addressed (among other things) the geological evidence cited to demonstrate that the Earth is 4 billion years old. Part II will deal with evidence from thermodynamics regarding the age of the universe. Note that this essay was written before the ramification of Edwin Hubble's 1929 law on cosmic evolution was appreciated. Hubble's Law is generally cited as the primary form of evidence for the Big Bang model of the universe. When addressing the age of the universe, contemporary scientists

It is known that some natural processes are reversible and others are irreversible.[37] Most mechanical processes, for instance, are reversible (if we do not take into account the process of friction). What is an example of an irreversible provess? Consider two adjacent rooms with a door between them. One room is hot and the other cold. If we open the door in the middle, the air would begin to move and an exchange would begin. This process, however, is irreversible; when the temperature in the two rooms equalizes the process will not revert on its own to its earlier state (unless we add energy to heat or cool the rooms) in which one room was hot and the other cold.[38] (This is what prevents the creation of perpetual motion machines of the second kind.[39]) This phenomenon which is well understood in practice is an enigma in theory; why don't the air molecules return on their own to their original places where they were before the door was opened?[40] Is there any law in nature that prevents them from doing so? The answer is that there is no [deterministic] *law* [חוק], but there is a *probability* [אומדנא]; irreversibility is a state of probability so high that approaches a [deterministic] law [אומדנא הקרובה לחוק]. To clarify, every molecule of air moves freely and reversibly according to the laws of mechanics. In principle then, and in theory, millions of molecules could move in the

typically refer to Hubble's Law which suggests that the universe had a beginning and is about 15 billion years old, rather than appealing to geological evidence about the age of the Earth.

37 Cf. (Fraenkel 1925).

38 Fraenkel is referring to Poincaré's Recurrence Theorem. This celebrated mathematical result says that if there are two rooms with a door separating them and all the molecules are in one room, and then the door is opened, if collisions of molecules in the two rooms are elastic (i.e., we assume there is no friction), then the molecules will first equalize into the two rooms, then eventually all return to the original room, then equalize into the two rooms, then return to the original room… *ad infinitum*.

39 See footnote 42.

40 This is well understood in practice because we know that the temperature in the two adjacent rooms always equalizes and more or less stays that way. But in theory, we have Poincaré's recurrence theorem (see footnote 38) to contend with, that tells us that the molecules in the rooms revert to their original state over and over again. (For a popular exposition see (Ekland 1988), 38–40).

exact same direction, but in practice, when there is an abundance of molecules, we can estimate in advance a clear and definite tendency of the movements of the molecules with a probability that approaches law, that would equalize the temperature in the two rooms and prevent this process from being reversed. Similarly, if a game of roulette was to last long enough, red and black would eventually come up about equally often. (The "practical difference" [נפקא-מינא] between law and probability that approaches law is that, if a certain external force is found, which could separate the fast from the slow molecules,[41] it would be possible for that force to overcome the probabilistic situation without disobeying physical law. It could bring about such processes which in fact appear to be practically impossible.[42] By way of example, consider a heavy body lifting itself using the motion of its own molecules.[43] Incidentally, this is one way to understand how miracles can be brought about without violating, in principle, the physical laws of nature.[44])

The second law of thermodynamics states, in the probabilistic form just discussed that heat energy continuously grown at the expense of remaining forms of energy in the universe (e.g., mechanical, light, electric). If we thus accept the demand of materialist scientists that we apply our familiar physical laws to the universe as a

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41 I.e., the hot from the cold.

42 Fraenkel is describing what is known as "Maxwell's Demon." Maxwell's Demon is a thought experiment first described by the Scottish physicist James Clerk Maxwell in 1867 to show that the second law of thermodynamics is only a statistical law and not a deterministic law of nature. It shows this by describing a hypothetical violation of the law. The "demon" is a force which stays at the door and sorts the moving particles by separating the fast from the slow. It keeps the slow (cold) particles in one room and allows the fast (hot) particles to pass through the door.

43 All atoms in a body move about randomly in all directions. This is called "Brownian motion." All the atoms bounce off each other and ricochet randomly. In theory, each of the numerous atoms could end up, suddenly and simultaneously, going upward together. If that happens the body will lift itself up. However, the probability of this happening is so low that there is what Fraenkel would call a "probability that approaches a law" that tells us that it will not happen.

44 Laws are not violated as God acts as Maxwell's Demon bringing about events that are, theoretically, possible, but have statistically low probability.

whole and to the indefinite past, we are forced to conclude, by the second law of thermodynamics, that it is impossible for the world to have always existed. In an infinite amount of time all energy forms in the world would have had to be converted to heat energy and the universe would have died a "heat death."[45]

In the previous example in which the differing temperatures of the two rooms become equalized, we explained that in addition to the [reversible] mechanical laws that govern the particles there are also other, irreversible laws, in probabilistic form, affecting them. These latter laws do not contradict the former but rather coexists with them as a second layer of law. Hence it is possible that there are other influencing forces, besides those that are known to scientists, which do not interfere with the laws that govern nature. Professor Dessauer explains this elegantly in his beautiful book *Life, Nature, Religion*.[46] He gives a parable of a non-musical scientist observing a musician playing a harp.[47] The scientist confirms that what is produced by the musician is in accord with physical law, such as the law of energy equilibrium, etc. The scientist is able to describe the frequency of the tones, but he us unable to explain the tone-intervals and the compounding of certain notes. While the mu-

[45] The idea that the universe will die a "heat death" has its origins in speculation in the 1850s about the second law of thermodynamics. It was theorized that given enough time, all the mechanical energy in the universe would dissipate as heat energy. So if the universe is infinitely old, all energy should have already dissipated. Recall, that in light of Hubble's Law, scientists now believe that the universe had a beginning at the "big bang". So Fraenkel gives us a more or less concurrent proof that the universe was indeed finite.

[46] (Dessauer 1926). Note (Fraenkel 1925) with the same title. In (Fraenkel 1967, 184) he mentions Dessauer's book's influence on his father, to whom (Fraenkel 1925) is dedicated. He also notes that the author was a left-wing member of the Catholic German Center Party in the German parliament who lost his professorship and ended up being declared a "non-Aryan" in exile in Switzerland after the Nazis took power.

[47] Fraenkel's footnote: A similar example was told over in the name of the Ba'al Shem Tov, of blessed memory, about Ḥasidim. Translator's note: "The Ba'al Shem Tov" is the moniker of Rabbi Yisroel ben Eliezer, the founder of the Ḥasidic movement. Fraenkel may be referring to the story in (Buber 1947, 54).

sician detects the melody, the scientist sees only random notes; in fact, to him, as a scientist, they must be interpreted as random for he believes he has a full understanding of all the phenomena involved. He rationalizes [יתפלפל] and says "if something escapes my notice, then the energy [equations] would be mismatched. But I already confirmed that all the energy is accounted for." And it does not at all occur to the scientist that there could be aspects of these phenomena—a musical melody, or some vital principle, which do not interfere with physical law but which, in fact, establish it and provide it with purpose. Similarly, we can imagine that the various laws of nature are, in actuality, manifestation of factors that are outside the bounds of science and beyond the course of nature; yet it is, even in principle, impossible for us to investigate the reason for these types of factors via scientific observation or to recognize their impact on the hubbub of nature. Essentially, natural science can only tell us about those forces and processes that can be captured by the methodology of natural science. Moreover, since the hubbub of nature and its laws cannot be completely grasped, we may conclude that there are other spiritual and godly forces that do not depend on science. There are higher, hidden laws that are layered above the known laws.

To summarize: in light of recent developments in the natural sciences, especially in light of modern physics, we see on the one hand the impermanence of the concepts and assumptions that once seemed timeless, strong, and irrefutable; on the other hand we also see that the words of Torah are not harmed or refuted by them, and it is our responsibility to hold on with an artist's faith, to the words of the sages: "Turn it over and over, for all is in it."[48] Or the words of the poet Rabbi Judah Halevi regarding Israel:

אַךְ שָׁמְרוּ אֱמוּנוֹת   בָּם הָיוּ אֱמוּנוֹת,
וְלֶאֱלִילֵי תְמוּנוֹת   מֵאֲנוּ לֵעָנוֹת. [49]

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48 *Chapters of the Fathers* 5:21.

49 This is a line from one of Judah Halevi's poems of redemption. See (Zimorah 1964) pp. 284ff. This fragment may be colloquially rendered: "Only guardians of the faith, they were trustworthy. But to pictures of gods, we refuse to be humbled." This evokes Fraenkel's thesis: remain faithful, and do not be overly concerned with the "pictures" of science.

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