The Pool of Shlomo HaMelech and the Value of π[1]

By: MORRIS ENGELSON

Foreword

The value of pi based on Solomon's Pool (*Yam Shel Shlomo*) in I *Malachim*, 7:23 has a rich literature in Torah and general secular and mathematical sources. This article is not intended as a survey of this literature, and certainly not as a commentary on the extensive Torah-based analyses. My intent is to acquaint the Torah-focused reader with some of the many approaches to a resolution of the puzzle posed by the stated dimensions of the *Yam Shel Shlomo*. In particular, we will show how and why these approaches are deficient when these ignore information derived from Torah. Only an approach based on Torah, and especially one that considers the volume discussed in *Eruvin,* yields consistent results.

Defining the Issues

It would appear from a simple reading of the text that neither the people Yisroel nor their leaders at the time of Shlomo HaMelech nor *Chazal* over a thousand years later were aware that the ratio of the circumference to diameter of the circle, currently designated by the symbol π (pi), is greater than 3. Thus, we are provided the dimensions of Solomon's circular pool as being 10 *amos* (cubits) in diameter, 30 *amos* in circumference, 5 *amos* in height and with a wall 1 *tefach* (hand breadth) thick. The volume is given as 2000 *bas*. The same dimensions are also found in *Divrey HaYamim* (4:2). A *Mishna* in M*aseches Eruvin* (13b) provides information about circles:

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1 An alternate version of this article, focusing on the mathematics rather than Torah, will appear in the July 2017 edition of *The Journal of Humanistic Mathematics,* http://scholarship.claremont.edu/jhm/.

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"Whatever has a circumference of three *tefachim* has a width of a *tefach*." The text in both places clearly designates the value $\pi=3$.

This creates a puzzle because it is virtually impossible that these people were not aware that the result is more than 3. The *Bais HaMikdash* was built with precision and skill by experienced craftsmen. Surely Hiram (I *Malachim* 7:13) and his master builders knew of approximations to pi that had long been established by measurement and observation. We know that the Egyptian and Babylonian approximations for pi (3.16 and 3.125 respectively) were established some five centuries before the time of Shlomo HaMelech. Ordinary people might have been ignorant on this matter, but how is it that *Yirmiyah*, the author of *Malachim*, knew nothing about this?

Furthermore, by the time of the Talmud over a millennium later, knowledge of pi was much improved due to mathematical analysis. Archimedes (ca. 225 BCE) had centuries previously established the relationship of $223/71 < \pi < 22/7$. Moreover, numerous people were involved in Talmudic discourse over a period of over 200 years. It is impossible that some of these people did not know that π is not equal to exactly 3. So why did they retain the statement that a circle with circumference equal to 3 has a diameter equal to 1?

What is going on here?

There are claims that this provides evidence or even proof that the Torah is wrong and *Chazal* were ignorant of basic mathematics. A more sensible understanding is that the Talmud is not a mathematics primer; it is a Torah compendium. Furthermore, we need to consider the information in context. There are three levels of explanation. There is the simple straightforward meaning; an advanced explanation derived from a hint; and a hidden explanation embedded in a secret. The ordinary meaning is one that people ordinarily understand and use. The advanced explanation requires expert knowledge and is for practical application by expert craftsmen. The final level is hidden in a secret which may have to wait many years to be uncovered as other knowledge becomes available.

The Ordinary Meaning

The Talmud teaches Torah not only on a theoretical level, but also on a practical level for daily usage. What value of pi do we need for ordinary, common daily use? The complete value of pi is not available to us. This is because pi is an irrational number and is impossible to write down its exact value no matter how many decimal places or fractional designation one chooses. Whether we choose $\pi=3$, or π=3.141592653589793238462643383279502884, or pi to a billion decimal places, all are approximations. Elishakoff and Pines put it this way in: *"Do Scripture and Mathematics Agree on the Number π?"* [2]

> How good is good enough? Even the 1.2 trillion-digit approximation of π made by Professor Yasumasa Kanada of Tokyo University in 2002 is still only an approximation. It is humbling to realize that there is something that we can never really know, and π provides us with this experience.

π = 3 is the simplest approximation. This introduces an error of just less than 5%. Is that acceptable? It depends on the approximation's purpose. Today we usually designate π=3.14 for common usage. But would this approximation be useful without the aid of a calculator? Try finding the product of 7 and 3.14 without a calculator and modern writing implements. Compare that approximation with the simple 7x3=21. Is this simplicity worth the 5% error? Many would say it is. Indeed, *Chazal* appear to agree that 5% is an acceptable error. In *Maseches Succah* 8a, a 5.6 diameter circle is approximated by a circle diameter of 6; this is a discrepancy of 7%. The Gemara implies: "When can we say that an Amora was imprecise? When the approximation is small."

One can delve deeper into this matter. Indeed, Tsaban and Garber[3] claim that π = 333/106 is embedded within the Talmud, and likewise we have an excellent approximation for the irrational square root of two as well.

There are good reasons to accept that the ratio 3:1 was chosen for practical and *halachic* purposes. This should be sufficient to close the matter. But there are those who insist that there are hints within the description of Solomon's pool that yield a credible approximation to the value of π. The bulk of this article is intended to examine this claim.

A Hint to a More Accurate Value

The simplest analysis in this mode is based on the premise that the 10 *amah* diameter is the outside diameter of the cylindrical pool, while the 30 *amah* circumference is the inside of the pool. This yields a result for pi that is greater than 3. How much greater depends on some assumptions. Un-

2 *B'Or HaTorah* 17 (5767/2007) http://u.cs.biu.ac.il/~tsaban/Pdf/Elishakoff Pines.pdf.

3 "On the Rabbinical Approximation of π," *Historia Mathematica* 25 (1998) http://u.cs.biu.ac.il/~tsaban/Pdf/latexpi.pdf.

fortunately, no matter what assumptions are chosen, all suggestions encounter difficulties either with the ratio of the *amah* to the *tefach* or to the volume.

The website for St. Andrews University in Scotland proposes a very accurate value for π. The content's author, Bob Graf, correctly notes that we do not know the length of the *amah* in modern units of measure. He chooses an *amah* of 17.75 inches. This is the lowest possible value. Likewise, the size of the *tefach* is also unknown, and the author chooses a value of 4 inches, which is near the upper end of estimates. This makes the outer diameter of the cylindrical pool 177.5 inches, and the inner diameter 169.5 inches. The inner circumference is 30x17.75 = 532.5 inches. And π = 532.5/169.5 rounded to 3.141593; precise to 5 decimal places. But this yields an *amah* to *tefach* ratio of 17.75/4 = 4.4375, while the relationship is 6 for the standard *amah* and 5 for the small *amah*. This idea does not work.[4]

The above is a simple example of proposed solutions to the puzzle. Possibly the most extensive survey of analyses from a mathematical point of view that also includes much material from Torah will be found in: Andrew J. Simoson, "Solomon's Sea and π."[5]

The paper has extensive illustrations and mathematical equations for a wide array of shapes proposed for the pool. Simoson accurately quotes and references several sources from Torah in support of the various suggestions. Likewise, there is an extensive bibliography to various suggested solutions to this puzzle. This article is not meant to replicate the work of Simoson, and anyone interested can consult his paper. It should be noted, however, that while Simoson discusses the volume of the pool in units of *bas*, he does not deal with the equivalent in cubic *amos*. Additionally, some of the suggestions convert the dimensions of the pool into current units of measure based on assumptions of the lengths of the *amah* and *tefach*. Hence, we find that the results, some of which are most ingenious, fail the volume standard given to us by *Chazal*. In addition, while quite extensive, the survey is not exhaustive. Several clever suggestions exists that Simoson does not elaborate on or even mention. We will deal with two of these below. But it is first necessary to introduce the matter of volume.

A Matter of Volume

We have proofs in *Maseches Eruvin* page 14b that the volume of the pool was 450 cubic standard *amah* (CSA), of 6 *tefachim* to the *amah*. This is the

4 Comments about this idea appear at http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/other_links/Graf_theory.html.

5 *The College Mathematical Journal*, vol., 40, no 1, January 2009. https://jeremy-brown-vpk4.squarespace.com/s/Solomons-sea-and-pi-2009.pdf

same as 150 *mikvah* volumes at their minimal volume of 40 *seah* each [at 3 CSA per 40 *seah*]. The key point for us is that the volume should be 450 CSA. The Talmud quickly concludes that the volume of a cylinder of the given dimensions (assuming the diameter is on the inside) will yield only 375 CSA (using $\pi = 3$). Thus, *Chazal* conclude that the bottom 3 *amah* of the pool were in the shape of a square whose volume is 300 CSA, while the upper 2 *amah* is a cylinder whose volume is 150 CSA. The total volume is the required 450 CSA.

The above calculation goes further than simply stating that we use $\pi=3$ for ordinary usage. The statement, "$\pi=3$" is simply an approximation that most people can easily understand and accept. But claiming that a cylinder with a diameter of 10 *amos* and height of 2 *amos* has a volume of 150 CSA implies that π actually equals 3. Thus, there are those who argue that *Chazal* were ignorant of basic mathematics. This is obviously nonsense to anybody who learns *Eruvin* or other areas of Torah and sees the sophisticated mathematical analyses involved. *Chazal* are focused on *halachah*; mathematics is invoked only as an aid and not in its own right. No one could possibly claim, for example, that *Chazal* were not aware that a gap, which is *halachically* treated via *lavud* as if it did not exist, does in fact exist physically. The same applies here.

This article, however, is not focused on *halachah* but rather on the mathematical implications resulting from the dimensions of the *Yam Shel Shlomo*. Hence, we are obliged to not only show a value of pi greater than 3, but we need to do so with a shape whose volume is 450 CSA. The simplest resolution is along the lines proposed by Graf, as previously discussed, where the diameter of the outer part of the cylinder is equal to 10, while the inner diameter of the cylinder is less than 10. This can result in a value for pi precise to as many decimal places as one might wish. Unfortunately, the result violates various restrictions imposed by *Chazal*.

Simoson discusses several possibilities. Here are some examples in addition to the many ingenious solutions discussed by Simoson.

The hexagonal pool solution. The verse in *Malachim* states that the pool's top lip was in the shape of a lily flower. This flower has six petals, which some take to mean a hexagon. A regular hexagon whose side is s=5 *amos* will have a circumference of 30 *amos* and a maximal diameter of 10 *amos*. This fits perfectly the ratio of 3:1. The area of a regular hexagon with side length s=5 is given by $[(3\sqrt{3})/2]s^2 \approx 64.95$, and we do not get the required volume of 450 CSA for a height of 5 *amos*. This solution does not work based on the position of the Talmud that the volume of the pool is 450 CSA.

The flared lip solution. This solution is discussed in a general fashion by Simoson; here is a more detailed analysis. *Malachim* states that the brim was "like the lip of a cup." This analysis is based on the idea that top of the pool was flared and the bottom cylindrical, making the diameter on top larger than the rest of the pool. Peter Aleff argues on his web site that "the rim was flared," and elaborates in his book *Ancient Creation Stories told by the Numbers* in a chapter on "The old myth of King Solomon's wrong pi." The essentials of this analysis are available on the internet, where Aleff quotes the simplified pi formula from *Eruvin*: "that which in circumference is three hands broad is one hand broad." He comments that "scholars of the Enlightenment era were glad to concur with that interpretation because it allowed them to wield this blatant falsehood in the Bible as an irresistible battering ram…" But Aleff disagrees with Enlightenment conclusions.[6]

Here the 10 *amah* diameter is measured across the flared top, while the circumference is measured on the outside lower cylindrical body. Referencing various works on archeology and ancient science/mathematics (e.g., van der Waerden: *Science Awakening, Egyptian, Babylonian and Greek Mathematics* and Leen Ritmeyer: *The Temple and the Rock*), Aleff provides a well annotated argument for a fairly accurate value of π. He uses 7 *tefachim* to the *amah* based on archeological analysis and Egyptian units. The bottom of the pool is taken as one *tefach* thick, and this is deducted from the 5 *amah* height. Unlike other calculations that ignore this feature, the required volume of 2000 liquid measure *bas* is accounted for in the calculations.

Aleff demonstrates that it all works out perfectly. His calculation of the volume after, accounting for the thickness of the bottom and the flared lip is 304.04 cubic *amah,* which he equates to 2000 *bas*. This appears to be contrary to the position of *Maseches Eruvin* where 2000 *bas* is equated to 450 CSA. But it is not that simple, because Aleff uses a super large *amah* of 7 *tefachim*. The volumetric ratio to the Talmudic *amah* of 6 *tefachim* is $(7/6)^3$ and the two volumes can be made to agree by a judicious choice of the volume lost to the flared rim. The only issue is the choice of 7 rather than 6 *tefachim* to the *amah.*

Solution by the Ralbag. The Ralbag proposes a solution that on the surface appears identical to that proposed by Graf; the diameter is for the outside surface of the cylinder while the circumference is on the inside. But Ralbag, who in addition to his greatness in Torah was also a world-class scientist and mathematician, does not make the error that Graff

6 Based on the analysis available at http://www.recoveredscience.com/const100solomonpi.htm.

makes. His *amah* has 6 *tefachim* and he is mindful of the required volume at 450 CSA. He proceeds with the shape proposed by *Chazal* with the 3 bottom *amos* in a square and the top 2 in a cylinder. There are several ways to calculate the volume of the shape chosen by Ralbag. The best result, noted in the paper below, shows a volume of 446.8 CSA, and Ralbag states that his result is approximate; that is, he does not ignore the need for 450 CSA. An analysis of the position taken by Ralbag is provided by Shai Simonson, of the Department of Mathematics and Computer Science in Stonehill College, in *The Mathematics of Levi ben Gershon, the Ralbag.*[7]

Ralbag is not the first Torah authority to suggest a good approximation for pi embedded within the shape of this pool. We have the value $\pi = 3^1/_7$ in a book on mathematics, *Mishnat ha-Middot*, attributed to Rabbi Nehemiah (ca 150 CE). Elishakoff and Pines discuss this book in some detail.

A Summary and New Suggestion

I previously noted an objective to show a value of pi greater than 3, but also with a shape whose volume is 450 CSA. None of the examples discussed in this article or its references meet this objective. Even Ralbag, with his great erudition in Torah and mathematical skill, falls short on the volume. I propose two possibilities for pi which fulfill the volume requirement.

The two approximations for pi which come immediately to mind are the ancient Mesopotamian approximation at 3.125 (3+1/8 on clay tablet from Susa) and the ancient Egyptian approximation of near 3.16 (per Rhind Papyrus). Below are two suggestions which work well mathematically, but introduce numerous questions. Neither is a credible candidate for the pool's actual shape, but do obtain the called-for volume and have pi set at a known ancient approximation. In summary: none of the proposed solutions meets all of the called-for parameters.

Let pi equal 3.125. This early Babylonian value for pi is dated CA 1900–1680 BCE. We will use a variation of one of the suggestions in *Eruvin* 14b where the bottom 4 *amah* are rectangular with a volume of 400 CSA and the upper *amah* is circular with a volume of 50 CSA. The 50 CSA is achieved by mixing the standard *amah* at 6 *tefachim* with the small *amah* at 5 *tefachim*. The upper, circular *amah* is 6 *tefachim* high with an outside diameter of 10 small *amah* (at 5 *tefachim* each) and an inner circumference of 30 small *amah*. The wall thickness is one *tefach*, and hence the inner diameter

7 Available at http://u.cs.biu.ac.il/~tsaban/Pdf/MathofLevi.pdf.

is 48 *tefachim*. The circumference is 30x5 = 150 *tefachim*, and π = 150/48 = 25/8 = 3.125.

We calculate the volume of this short cylinder as follows: the inside diameter is 48 *tefachim* with a wall thickness of one *tefach* which is 8 regular *amah*. The square of the radius is 16 and with π at 3.125 we get a volume of 50 CSA. The total volume is the required 450 CSA. This ancient Mesopotamian approximation for π was discovered by Otto Neugebauer on a clay tablet, as discussed in his *The Exact Sciences in Antiquity*.

There are a number of issues here, the most important being the mixing of different size *amos*.

Let pi equal 3.16. This approximation for pi comes from the Rhind Papyrus which is dated to 1650 BCE. Pi is shown to be 256/81≈3.1605. Here we use a variation of the choice by *Chazal* where the lower three *amos* are rectangular while the upper two *amos* are circular. The procedure follows in the footsteps of Ralbag where the 10 *amah* diameter is on the outside while the 30 *amah* circumference is on the inside, resulting in a value for pi greater than three. But we want pi not only greater than 3, but specifically near 3.16. Thus, we adjust the wall thickness to be not one *tefach* but 1.5 *tefachim*, yielding an inner diameter of 57 *tefachim* (10 *amos* = 60 *tefachim* – 2x1.5). The value of pi is 180/57 (*tefachim*) = 3.15789…; approximately 3.16. We now adjust the lower 3 *amos* in a square to be 3.25 *amos* high and the upper two cylindrical *amos* to be 1.75 *amos* tall. The lower 3.25 *amos* yield a volume of 325 CSA. The radius of the cylinder is 57/2 *tefachim*, which is 57/12 *amos*, and the area is the square of this radius times pi (3.16), or almost 71.3. The area times the height of 1.75 yields a volume of 124.77…, which we round to 125. The total volume is 325+125=450 CSA.

One issue is the designation of a wall thickness at 1.5 *tefachim* when the text in *I Malachim* specifies 1 *tefach*. We justify this based on the conjecture that the measuring rope (*kav*) was used to measure the 10 and 30 *amos*, but the one *tefach* wall thickness was based on a visual approximation. We are told how the round shape and dimensions with diameter and circumference are measured, but are given no indication that *tefach* wall thickness was measured. Could the thickness have been an estimate? I built a scale model and drew a portion of the circle on the ground to scale. It was difficult to make an accurate estimate, however, the rim thickness is clearly not an *amah* or *etzbah*. It appears to be somewhere near a *tefach*. I found it difficult to differentiate between 1 and 1.5 *tefachim*.

Hence, I would argue that it is possible that the wall thickness may have been as much as 1.5 *tefachim*.

A Solution Based on Secret Knowledge

This solution is attributed to Eliyahu the Gaon of Vilna (18th century), known as the Gra. The Gra was not only a *Gadol* in Torah, but also an accomplished mathematician. However, there is evidence that this solution was first proposed by 20th century Torah scholar, Rabbi Matityahu Hakohen Munk (Max Munk), as noted in a private communication to me by Rabbi Professor Sid Leiman of Brooklyn College. He states: "The Gra did talk about pi, but never suggested the secret interpretation ascribed to him. That interpretation was first suggested by Max Munk in 1939. He published his suggestion in "Shalosh Ba'ayot Handasiyot be-Tanakh uve-Talmud."[8]

The result uses an (alleged) secret hidden in the spelling of the Hebrew word *kav* (measuring rope) with which the dimensions of Solomon's pool were determined. The *gematria* (numerical value) of the standard spelling of *kav* has the value 106. But the section in I *Malachim* has *kav* spelled with a superfluous letter *hei*, whose numerical value is 5. Hence this *kav* has the numerical value 111. We now apply the correction factor 111/106 to the ratio 3 given in the text, and get π precise to four decimals at 3(111/106) ≈ 3.1415. The purpose of the extraneous *hei* would be unknown even to the author (attributed to the prophet *Yirmiyah*) who would have used this variant spelling on the basis of prophetic knowledge. But progress in mathematics eventually lead to recognition of the significance of the extra *hei* and the secret was revealed.

This factor of (111/106) x 3 = 333/106 has some interesting mathematical properties as discussed in *The Bible and Pi*.[9] Deakin & Lausch provide a detailed analysis, including use of Hebrew letters when discussing *gematrios*.

The authors derive several fractional approximations for pi. Thus, in order of increasing precision: $\pi_o = 3$, $\pi_1 = 22/7$, $\pi_2 = 333/106$, $\pi_3 = 355/113$ … $\pi_5 = 104348/33215$, etc. The analysis then states that

> The surface meaning of the text gives the value π_o, but this is deceptive. Those in the know (so the story goes) see hidden in the text the more accurate value π_2. Now either the Rabbinical tradition is responsible for π_2, and the author of 1 Kings surreptitiously coded into his text an extremely accurate value of π, or else we have a most remarkable numerical coincidence.

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8 *Sinai* 51(1962), 218–227.

9 Deakin & Lausch, *The Bible and Pi*, The Mathematical Gazette, July 1998 (Vol 82.494). Available at http://poncelet.math.nthu.edu.tw/disk5/js/math-gazette/bible-pi.Pdf.

The "most remarkable numerical coincidence" notwithstanding, Simoson, quoting Deakin & Lausch, argues for a coincidence. Thus:

> Deakin points out that if the deity truly is at work in this phenomenon of scripture revealing an accurate approximation of π, a much better fraction not far removed from 333/106 would most definitely have been selected instead.

The proposed choice is 355/113 and we are given several reasons why it should be so. This is certainly a good choice from a mathematical point of view, but virtually impossible from a practical or operational point of view. Remember that the objective is not only to provide a more accurate value for π but also to have the information hidden. Otherwise the author could have simply provided us with a more accurate value to start with. The key point is that we must start with $\pi = 3$ and the hidden knowledge is a correction to that value. But no correction is possible by multiplication as for 333, since 3 is not a factor of 355. A correction could possibly be contrived mathematically by judicious addition, but what Hebrew words or phrases could be found to surreptitiously introduce this value which needs to be in the form of a fraction? I conveyed the counter argument to Professor Simoson via email and he agreed that it is a valid point.

There is yet another argument against 355/113 which I noted earlier in the discussion of the proposal by Graf. We could not indicate this previously for lack of background, but Graf did not begin with 17.75 and 4 inches. These values were derived from the 355/113 approximation to pi. Indeed, 355/113 is an excellent approximation for π, but unfortunately this yields an impermissible ratio of *amah* versus *tefachim*. There is no better result than 333/106 that works on a practical level.

Simoson also brings up possibly the most obvious objection:

> A natural question with respect to this method is, why add, divide, and multiply the letters of the words? Perhaps an even more basic question is, why all the mystery in the first place?

"The mystery in the first place" is within the inherent nature of π, which cannot be directly expressed by any written number. How does one provide a "true" value without teaching the advanced mathematical concepts inherent in the meanings of irrational numbers? Furthermore, how does one provide a simple and usable approximation while also indicating that there is much more involved? Even a novice learns quickly that the same overtly simple *posuk* is understood with far more complexity by his *rebbe* than his own seemingly complete understanding. The depth of Torah is infinite; it is up to the task of dealing with the multiple facets of pi.[10]

Belaga introduces a number of questions and suggests answers respecting this approach. Here are two additional factors.

The extra *hei* appears three times – in I *Malachim*, and also *Yirmiyah* 31:39 and *Zechariah* 1:16. A reviewer suggested that the 3 places with an additional *hei* are intended to call our attention to the base number, 3, to which we apply the correction 333/106.

Also, despite the above, there is an interesting positive point to consider. Taking 106 as fixed and given the normal spelling of *kav*, to give us a good approximation of π, the second number should be 111. In fact, we need to go three more decimals to 111.003 before obtaining a more precise result.

Conclusion

Which of the above three levels of explanation is correct? There is no way to prove any. However, all three levels could be correct, as the Torah is simultaneously structured on different levels of knowledge and understanding. The simple value 3 exists to help us make practical decisions in ordinary circumstances. It was especially useful to people several thousand of years ago when calculation of areas and volumes was a major task. The values 3.16 and 3.125 are in line with the scientific and mathematical knowledge of the time and would have been used by skilled craftsmen of that time. Finally, we have a hidden result based on prophetic knowledge which is in line with our time and our mathematical sophistication.

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10 A detailed analysis on the history of this approach, including additional references such as to Rambam on E*ruvin* (*Mishnah* 1-5) where he appears to state that pi is irrational, appears in *On The Rabbinical Exegesis of an Enhanced Biblical Value of π* by Shlomo Edward G. Belaga, available at http://www.math.ubc.ca/~israel/bpi/bpi.html.

Addendum

The following provides explanatory matter.

Why did Ralbag ignore my shape for pool?

Ralbag uses the same basic shape for the *Yam Shel Shlomo* as *Chazal* with 3 *amos* square and 2 *amos* circular, but changes the mode of calculating the value for pi and the resulting volume. This yields a value for pi greater than 3, but a volume less than 450 CSA; all such procedures will yield a volume less than 450 CSA. His effort appears to be to favor a mathematically more acceptable value for pi at the expense of the volume.

I introduce a shape that is somewhat different than introduced by *Chazal* and used by Ralbag. The value of pi is greater than 3 and we achieve the exact required volume. Ralbag was a world-class mathematician and published a book on geometry. It is inconceivable that he could not devise the shape of the pool I use. Given that my shape yields a closer approximation to pi than his shape and also the exact volume, why did he not choose my shape? There are several possibilities for why Ralbag would have rejected my proposals. My shape differs from the one chosen by *Chazal* in three respects:

(A) I have 4 *amah* in a square and 1 *amah* circular rather than 3 and 2 respectively for *Chazal*. Both versions appear to be acceptable and the choice was made simply on the basis of which shape introduced the appropriate volume. The issue is not the shape but rather the choice for the value of pi. *Chazal* choose pi=3 and this determines which shape to use. But both Ralbag and I choose to emphasize a value for pi closer to the correct mathematical value at more than 3. Surely Ralbag would not object to this.

(B) *Chazal* and also Ralbag stay throughout with the standard *amah* at 6 *tefachim*. I have both the standard *amah* at 6 *tefachim* and also the small *amah* at 5 *tefachim*. I believe that this is the most serious objection to my proposal. I know of no reason why this should be prohibited given that, while highly unusual, there are cases of mixing *amah* sizes in the *Bayis Rishon*. It is clear that Ralbag chose to emphasize the value of pi over other considerations such as achieving the exact volume. Would the mixing of different size *amahs* be sufficient for him to disallow a shape that introduces a good approximation for pi?

(C) *Chazal* focus on the inner diameter in their calculations while I sometimes use the inner and sometimes the outer diameter. But Ralbag employs equivalent methods.

Given the above, it appears to me that the likely position is that he did not use my shape for whatever reason that may be, but not because that shape is prohibited by Torah. There is a possibility, though, that Ralbag would decline to use any shape that is not expressly chosen by *Chazal*, even if this shape is not directly prohibited.

The other possibility is that Ralbag did not know of my shape. He did not have my advantage of knowing to search for a value of pi at 3.125 based on what the skilled craftsmen of the time knew. 3.125=25/8 calls attention to a diameter involving the small *amah* with one *tefach* wall thickness.

I don't know why the Ralbag didn't search more vigorously for a shape that would have satisfied both the pi approximation as well as the volume number. Ralbag seems to have compartmentalized his Torah and mathematics. The position of Torah as provided by *Chazal* is that pi equals 3. No doubt Ralbag was satisfied with $\pi=3$ for purposes of Torah but was not satisfied with this value for purposes of mathematics. He apparently chose to emphasize the mathematical aspects of his calculation rather than the Torah aspects. Thus, for the sake of a better approximation for pi, Ralbag was willing to settle for an approximate volume level. Why?

We know that the Kohanim used the pool as a *mikvah*. The pool's volume apparently has no *halachic* implications. This is not as in the case of a regular *mikvah* which becomes invalid even if it is somewhat deficient from 40 *seah*. This pool's volume is listed as 150 *mikvaot*. This is not a *halachic* requirement; it is simply the volume derived from the stated dimensions of the pool. Ralbag uses a shape that yields just a bit less than 149 *mikvaos,* and was apparently satisfied with the statement that the result is approximate. Hence, I conjecture that he did not bother to search for another shape. And while Ralbag chose to emphasize the mathematical aspects of the shape, it might be that he preferred not to delve into the mathematics and away from the Torah more than necessary in order to satisfy the need for a good approximation to pi. Hence, sticking to a shape that is the same as that chosen by *Chazal* would be preferred to looking for a different shape.

It would be interesting to know what Ralbag would have done had he been handed the shape that I use where the approximation for pi is better than his and where we achieve the exact volume called for by *Chazal*, but with a shape slightly different than that which they chose.