

The Tallest Column: On Monetary Value of Stature in Jewish Law

By: ISAAC EHRENBERG, JOSHUA SIEGEL and DYLAN ERB

Introduction: Donations to the Temple: *Erechin* vs. *Damim*

In contrast to the strict requirements of *Erechin* pledges found in Leviticus 27:1–8, *Damim* (monetary) donations are not closely regulated by the Torah and can take many forms. One may not, for example, offer an *Erech* that corresponds to some fraction of a human being, as *Erech* donations can be made only on an integer number of persons. To use language that could refer to anything but the entirety of a person renders the *Erech* attempt failed (*Erechin* Chapter 5 Mishna 2). *Damim* is more flexible as the giver is free to offer almost any value, flexibly comprised. Beyond specifying a donation in the local currency, the Talmud quotes two cases where people offered either their entire value on the slave market, or the more complicated value of a particular body part such as a hand, which requires two evaluations to solve the equation ‘Value of one’s Hand’ = ‘Value of that Person’ – ‘Value of (Person without his Hand).’ A similar situation arises when one offers to contribute the value of his weight in a material. A simple measurement is needed when the entire weight is specified, but if it’s only the weight of a particular limb, the Mishna (*Erechin* 19a) details an elaborate procedure that involves measuring the volume of the limb through a water displacement exercise, and estimating the density of human flesh using a combination of donkey bone, blood and meat.

The Value of Stature

In the above situations there is clear Talmudic direction given to evaluate the monetary value of a particular pledge, but that is not always the case. Aside from pledges of worth and weight, the Talmud (*Erechin* 19a) records an Amoraic position regarding one who pledges “his Stature”:

Isaac Ehrenberg (corresponding author, yitzi@mit.edu) is an affiliate of the MIT Auto-ID Laboratory, a member of the technical staff at the Charles Stark Draper Laboratory and a RIETS graduate.

Joshua Siegel is a Research Scientist at the MIT Auto-ID and Field Intelligence Laboratories.

Dylan Erb is an affiliate of the MIT Auto-ID and Field Intelligence Laboratories, and is member of technical staff at Ford Motor Company.

All authors have PhDs in Mechanical Engineering from MIT.

אמר רב יהודה: האומר קומתי עלי נותן שרביט שאינו נכפף, מלא קומתי עלי נותן שרביט הנכפף.

Rav Yehuda states that one who says: “my Stature is upon me,” he gives a rod that cannot bend; “the fullness of my Stature,” he gives a rod that can bend.

The Gemara, taking no issue with the strange language of the pledge, instead focuses on R’ Yehuda’s distinction between קומתי (my Stature) and מלא קומתי (the fullness of my Stature), quoting a Beraisa that appears to equate the two terms:

מיתבי קומתי עלי מלא קומתי עלי נותן שרביט שאינו נכפף.

If he says “my Stature” or “the fullness of my Stature,” he gives a rod that does not bend.

The Gemara explains the two statements as coming from different sources. R’ Yehuda does not hold like the author of this Beraisa, who clearly gives no credence to the added term מלא. Rather, R’ Yehuda follows R’ Akiva, who makes inferences from added words as seen from a Mishna in Bava Basra (20a) that is quoted here:

הוא דאמר כר"ע דדייק לישנא יתירא דתנן לא את הבור ולא [את] הדות אף ע"פ שכתב עומקה ורומה וצריך ליקח לו דרך דברי ר"ע וחכ"א אינו צריך ומודה רבי עקיבא בזמן שאמר לו חוץ מאלו שאין צריך ליקח לו דרך אלמא כיון דלא צריך וקאמר לטפויי מילתא קאתי הכא נמי כיון דלא צריך וקאמר לטפויי מילתא קאתי. The Mishna states: all agree that when one sells a house, the wells and cisterns are not included in the sale. R’ Akiva and the Sages disagree, however, about the pathway to said water sources. The sages hold rights to the path are implicitly included, while R’ Akiva holds the rights are not included unless the non-sale of the water sources was explicitly mentioned in the relevant sale document. We see from this that, though the seller was not obligated to affirm the clause regarding water sources but did so anyway, he wishes to retain the rights to the path. Similarly, by the case in Eruchin, Rav Yehuda holds that if one included the term ‘מלא’, then we can infer its inclusion was intentional and meaningful, and thus קומתי and מלא קומתי are taken to mean different things.

One question that arises from the basic reading of this Talmudic passage relates the exact meaning of קומתי and מלא קומתי. While the words נכפף (bends) and שאינו נכפף (that does not bend) are descriptive, the literal meaning is somewhat vague for the purposes of a monetary evaluation. Rashi weighs in on the issue as follows: קומתי refers to a rod as tall as his height, while שאינו נכפף means (שרביט עב שלא יוכל לכופפו) the rod must

be thick such that it is unable to bend, and from a material of the pledger's choice. In contrast, מלא קומתי is a case where one specifies he is interested in donating a rod that has the length of his height, but no dimension of thickness is specified.

According to Rashi, the word קומתי could be said to define two parameters that specify the amount of material one intends to donate: the height and diameter of a cylindrically shaped rod. The height is equivalent to that of the donator, and the diameter is dependent on the cylinder's ability to bend. The extra word מלא then comes to qualify that only the height parameter is fixed.

Known for its terseness, Rashi's language is worth comparing with that of Rabbeinu Gershom, which is more expansive on this topic. Regarding the שרביט שאינו נכפף, R' Gershom explains it as a rod that is

עב שאינו נכפף מעצמו... כלומר דבר שיכול לקיים מעצמו שאינו נכפף.

Of thickness such that it doesn't bend from itself, meaning something able to support itself such that it won't bend.

Whereas Rashi's shortened language describes the rod as so thick it is "unable" to bend and he leaves the load under which there is no bending unspecified, Rabbeinu Gershom's language of שיכול לקיים מעצמו implies some minimal load threshold, and the existence of some critical thickness below which the scepter bends under its own devices. Despite any potential difference between these two opinions, the positions align for the case of מלא קומתי, where both agree the only specified dimension of the bendable rod is height, with Rashi saying the lack of specified thickness was purposeful, and Rabbeinu Gershom agreeing that even a thin rod is valid in such a case.

Several difficulties arise when considering the Gemara with the explanations of the Ashkenazi commentators while quantifying the money owed as a result of either donation formulation. Firstly, how can we relate the rod's diameter and its susceptibility to bending? While it's obvious that the force required to induce bending of some kind is dependent on material thickness, both the Gemara and the Rishonim leave the problem ill-defined. The language of Rashi is particularly troublesome, as his formulation seems to translate as "the value of a rod that has significant thickness such that it is unable to bend." Technically speaking, "unable to bend" cannot be an exact term, as even the most brittle material has a Modulus of Elasticity (Young's Modulus) and will bend under a load. Given this reality, even if we are certain that מלא קומתי and קומתי signify different amounts, we are left without clearly delineated boundaries as to what constitutes a bendable rod.

This point can be demonstrated by considering the example in Figure 1, which shows a cantilever beam. One end is free (A), and the other is fixed (B) allowing for no translation or rotation. The uniformly applied weight loading w is the result of the Earth's gravitational pull exerted on the cantilever of length L and radius r , with the cantilever consisting of a homogeneous material of an isotropic Young's Modulus E .

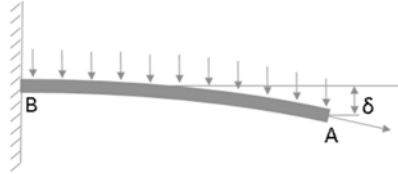


Fig. 1: A cantilever beam under the evenly distributed load w

Solving for the deflection δ of the neutral axis under this constant load gives

$$\delta = \frac{wL^4}{8EI}$$

where L is the length of the rod, and I is the moment of inertia of the cross-sectional area about the beam's neutral axis, which for a cylindrical rod is

$$I = \int \int z^2 dydz = \int_0^{2\pi} \int_0^r (r \sin \theta)^2 r dr d\theta = \frac{\pi r^4}{4}.$$

Given that

$$w = \frac{mg}{L} = \frac{\rho Vg}{L} = \frac{\rho \pi r^2 Lg}{L} = \rho \pi r^2 g$$

we can solve for the maximum deflection of the rod at point A in terms of rod radius:

$$\delta = \frac{\rho g L^4}{2Er^2} \text{ (Equation \#1)}$$

where ρ is the density of the rod material and g is acceleration due to gravity at the Earth's surface.

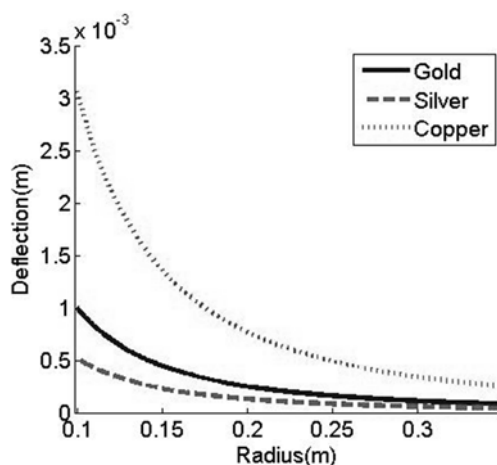


Fig. 2: Deflection of a 1.7m cylindrical rod of Gold, Silver and Copper for a given radius.

This makes clear that there is no possibility for zero deflection in the presence of the ubiquitous gravitational force. This analysis does however raise the possibility of defining a ‘bending/non-bending’ threshold as deflections become small such that it’s possible to place a bound on the amount of allowed deflection for a “non-bendable” rod. Such an approach could take cues from other areas of Halacha, such as allowable deviations tolerated in modern *Erums* (wires, doors, frames / *Tzuras Hapesach*) and issues that require objects to be visible to the naked eye (eating bugs in produce).

Given precedent to choose a point where any deflection could be ignored, what ends up being more troubling is the lack of a lower limit provided for the diameter of the bendable rod implied by מלא קומתי. As the enforceable value of one’s pledge amounts to the minimum expressed using that particular language, following the implied opinion of the Rishonim that the radial dimension is unbounded to its logical conclusion would result in the donation of a one-dimensional string of atoms or molecules, which would hardly be of any worth. Given that the Gemara indicates this bendable rod has value and is not microscopic, we are forced to search for other definitions to avoid this trivial solution.

We therefore consider another situation in the Talmud where an Amora gives a definition for the amount of material included in a donation. On Shabbos 90a and again on *Menachos* 107a, the same statement appears to specify the minimum donation of iron to the Temple:

ת"ר האומר הרי עלי ברזל אחרים אומרים לא יפחות מאמה על אמה למאי חזיא
אמר רב יוסף לכלייה עורב ואיכא דאמרי אחרים אומרים לא יפחות מכלייה עורב

וכמה אמר רב יוסף אמה על אמה.

The rabbis taught—If one pledges “Iron”: Some say you can’t give less than 1 cubit by 1 cubit, and what is this good for? R’ Yosef says for a Crow Chaser. Others say you have to give enough for a Crow Chaser, and how much is that? R’ Yosef says 1 cubit by 1 cubit.

Whichever version of the statement we decide to take, we come out again with an ill-defined amount of material. The thickness of the iron plate is not provided, and neither are schematics for the Temple Crow Chasers.

Ironically, while one puzzles about the lack of dimensions used to specify the iron’s volume, the Mishna (*Menachos* 106b) and Gemara seem to give more useful definitions when it comes to the minimum amounts that must be given in the event of an undefined pledge of precious metal. The minimum for gold and silver, for example, is the size of a Dinar. The Gemara explains, however, that this applies only when one specifies coinage. In the absence of legal tender, it appears that something smaller is permitted, a נסכא, which is translated as an ingot, literally meaning a hunk of cast metal. Lest the size of this נסכא donation also approach the weight of an atomic mass, Tosfos on 107a quote the Gemara in *Shekalim* (daf 11) and Shabbos 90a that the ingot should be large enough to form a hook, presumably the smallest amount that could be specified as ‘useful.’ So, there appear to be two categories when it comes to the minimum donation: for currency, one must give the smallest circulating coin. For materials, one must give the smallest useful amount.

One could argue that these criteria should be used to complete the dimensions of the 1amah x 1amah iron plates, and could apply to the rod pledged with the phrase מלא קומתי as well. A rod that can be bent has the same height as the donator, and a radius large enough to satisfy the Mikdash minimal-use requirement.

Following this formula makes מלא קומתי seem like a roundabout way of pledging the minimum useful donation. That minimum makes sense when the situation is like the case in *Menachos*, when the person just says one word like ‘gold.’ Though “the fullness of my Stature” is often used to specify length (*Shir ha-Shirim Rabbah* 4:7), it may be reasonable to assume the Gemara in *Eruchin* means something more.

a) The Tallest column that can support itself and will not bend.

We would like to propose physical definitions that may enable us to better define שרביט נכפף and שרביט שאינו נכפף for the Ashkenazi explanations of מלא קומתי and קומתי, without resorting to an arbitrary threshold for the definition of “unable to bend,” and without relying on the minimal useable volume bound for a rod that “can bend.” To do so, we suggest a

physical criterion that defines a piece of material as being a שרביט as opposed to something else, and within that definition of שרביט, define a way to differentiate between נכפף and lack thereof.

For the purposes of this Gemara, a שרביט, translated as rod or scepter, should be defined as a cylindrical object with a length significantly larger than its radius yet capable of supporting itself while fixed vertically without buckling under its own weight. An analytical expression that satisfies this criterion was developed in the year 1757CE by Swiss physicist and mathematician Leonard Euler.

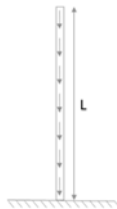


Fig. 3: A vertical rod loaded by its own weight.

Euler examined the problem of prismatic column buckling and found that a vertical column, fixed at its base and unsupported at the top as shown in Figure 3, has a critical length of:

$$L_c = \left(\frac{9EI}{4\rho gA} j_{-\frac{1}{3}}^2 \right)^{\frac{1}{3}}$$

where E is the material's Young's modulus, I is the moment of inertia, ρ is the density of the material, A is the column's cross-sectional area and $j_{-\frac{1}{3}}$ is the least positive root of the Bessel function of order $-\frac{1}{3}$.

Using a cylindrical cross-section as above, we find $A = \pi r^2$ and $I = \frac{\pi r^4}{4}$. The critical-length equation therefore becomes:

$$L_c = \left(\frac{9Er^2}{16\rho g} j_{-\frac{1}{3}}^2 \right)^{\frac{1}{3}}$$

From rearrangement of this equation, we can find a solution for the critical radius of a uniform-cross section rod:

¹ Cox, Steven and McCarthy, Maeve. "The shape of the tallest column." In Society for Industrial and Applied Mathematics, Volume 29, Number 3, pages 547–554. May 1998.

$$r_c = \frac{4}{3} \sqrt{\frac{\rho g L_c^3}{E(J^2_{-1})}}$$

This requirement for a piece of material to be called a שרביט places a reasonable lower bound on the radius of rods referred to by the Gemara.

If one is satisfied that a hook offers a minimum solution for bendable rods, this tallest column solution provides clear demarcation for what would be considered “a rod that doesn’t bend.” It would also fit into the words of R’ Gershom-דבר-שיכול לקיים מעצמו דבר, something able to support itself. However, having chosen to dispense with that minimum useable criteria and instead requiring both the bendable and unbendable rods to support themselves without buckling, another definition is still needed to delineate what is נכפף and what is not. Despite prescribing a minimum radius r_c for a rod of a given length L , the tallest column doesn’t change the fact that any rod subject to non-axial forces such as the fixed free cantilever of Fig. 1 will still deflect a fixed amount, which necessitates a redefinition of what it means to be a שרביט הנכפף, a rod that bends.

One possible clarification may differentiate between purely elastic bending and plastic bending. Elastic bending, assumed by Eqn. 1, refers to reversible strains or deflections. Plastic bending, on the other hand, refers to permanent deformations that occur when the stresses induced by bending cause strains exceeding the material’s elastic limit (“yielding”).

We can now define the שרביט שאינו נכפף as a beam of real material that deviates from linearity through elastic deformation but resists bending beyond the point of permanent plastic deformation. The only question is: what is the critical load? We again look to the wording of R’ Gershom, דבר שיכול לקיים מעצמו שאינו נכפף, and explain it as follows: שאינו נכפף means something that will not strain to yield and plastically deform from its own weight as a horizontal cantilever. Taking the rod in the same configuration as Fig. 1, we can solve for the smallest radius of a cantilever of length L for which there will be no plastic deformation,

$$r_c = \frac{2\rho g L^2}{\sigma}$$

Any rod with a radius less than r_c will plastically deform when fixed horizontally, and it is such a rod that we propose to define as הנכפף, as long as the rod radius remains above the threshold of what we defined as שרביט.

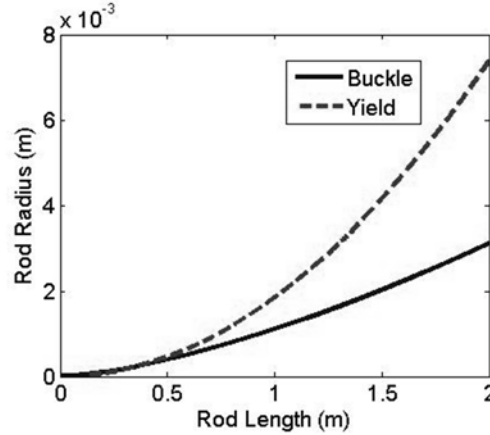


Fig. 3: Critical r_c dependence on length for a gold rod (blue) as a horizontal column such that it will not buckle under its own weight, and (red) as a fixed-free cantilever such that it won't yield under its own weight. The red curve corresponds to our definition of a שרביט שאינו נכפף, and the blue to our lower bound for the שרביט הנכפף.

While not without underlying assumptions, these definitions for שרביט הנכפף and שאינו נכפף provide well defined, realistic values for Rashi and R' Gershom that rely on material and environmental properties as opposed to otherwise arbitrary criteria that would be needed to differentiate rigid from bendable from infinitely thin rods.

Material	$r_{\text{הנכפף}}$	$\$_{\text{הנכפף}}$	$r_{\text{שאינו נכפף}}$	$\$_{\text{שאינו נכפף}}$
Gold	2.45mm	23,802	5.33mm	112,710
Silver	1.8mm	109	10.8mm	4,120
Copper	4.3mm	64	72.5mm	18,304

Table 1: Value of donations for a 1.7m tall person near current market prices.

Where this solution encounters difficulty is in its application to short individuals. Though it changes for every material, there is a point where the radius of the rod defined as שרביט שאינו נכפף is actually smaller than the minimum radius for שרביט הנכפף. Aside from no longer fitting into our definition of שרביט, in such a case the donation for קומתי becomes more valuable than that of מלא קומתי, which seems to be against the underlying assumptions of Rashi and R' Gershom.

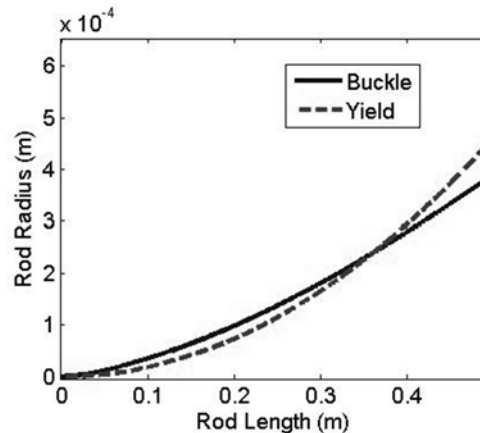


Fig. 4: The yield radius of the gold rod is smaller than the buckling radius when the length is below 36cm, which means these definitions for שאינו נכפף and הנכפף would not be appropriate for short people.

b) מלא Means More

Though both Rashi and R' Gershom take it for granted that the value of the “non-bending” rod is more valuable than one that “bends,” their assumption is brought into question by the Nezer Hakodesh, who is puzzled by the use of the word מלא to signify a pledge of lesser value. He counters that appending the word מלא to קומתי should at least reinforce, if not significantly enhance, the value specified by קומתי. He finds support for this opinion from what appears to be a variant textual tradition quoted by R' Gershom:

ואיכא דאמרי איפכא קומתי עלי נותן שרביט הנכפף מלא קומתי עלי נותן שרביט שאינו נכפף דכל לישנא (בתרא) [יתירא] לטפויי מלתא אתא.
 ‘My Stature’ obligates one to bring a rod that bends, and ‘the fullness of my Stature’ obligates one to bring a rod that doesn’t bend, as all extra words come to include something more.

While fitting this stance into the flow of the Gemara presents its own issues, it does lend credence to the idea that מלא קומתי should be worth relatively more.

Nezer Hakodesh notes that the standard Ashkenazi approach to our Gemara, namely the notion of the relative rod values specified by קומתי and מלא קומתי, comes from a particular reading of the Mishna quoted by the Gemara to elucidate R' Akiva’s view regarding the importance of added words. In that Mishna, the seller retains access rights to his water

sources by including extra terms in the contract. R' Gershom and Rashi explain the Gemara as follows: by reciting more words, the seller gives less. The same must be true by *קומתי* and *מלא קומתי*. If one adds a term, he is pledging to give less to the Beit Hamikdash.

However, one could suggest an alternative reading of this Gemara, the conclusion being that added words mean “something” additional, but the change of values need not move in the same direction as before. In the case of the property sale, the additional words signify the seller wants to retain assets and access to them. In a situation where one is donating money to the Temple by his own free will, it's reasonable to say a person's additional words may go in either direction, to increase or decrease the value of the pledge. In this particular case, the term *מלא* can be considered somewhat ambiguous. Rashi and R' Gershom may be taking their cues from other instances of *מלא קומתו* in the Torah or Rabbinic literature, though no specific reference is given, but it could be argued the phrase may not be common enough to discredit an alternative explanation.

c) Redefining *נכפף* and *שאינו נכפף*

In order to meet the Nezer HaKodesh's requirement of a more valuable bendable rod without following the variant version of the Gemara quoted in R' Gershom, we need not reevaluate the definitions of *קומתי* and *מלא קומתי*, but we will need to change the scenario in which the yield criterion is reached. As before, the definition of *שאינו נכפף* remains ‘unable to yield’ and the rod described by *נכפף* has reached yield, but the situation is no longer having to withstand the force of gravity. Instead, one possible test is whether the beam remains bent after being forced into a given radius of curvature under pure bending conditions, or if it remains elastic and achieves complete springback.

From compatibility ($\epsilon_{xx} = -\frac{y}{\rho}$), the definition of curvature ($\kappa = \frac{1}{\rho}$), the relationship between stress and moment ($\sigma_{xx} = -\frac{My}{I}$), Hooke's law ($\sigma_{xx} = E\epsilon_{xx}$) and the moment-curvature relationship ($M = EI\kappa$) we derive a relationship for the radius above which a cylinder will “spring back” to its original shape after being bent to a particular radius ρ :

$$r_{cyl} \leq \frac{\sigma_y \rho}{E}$$

where r_{cyl} is the cylindrical rod's bend radius, ρ is the radius of the forced bend, σ_{xx} is the yield stress (the point at which elastic deformation becomes plastic deformation at the outer fibers of a bent beam) and E is the rod's modulus of elasticity.

A cylindrical rod with a larger radius will result in some plastic deformation, and can be definitively labeled as a rod that is, and will remain, bent.

Though this gives us a reasonable distinction between שאינו נכפף and נכפף according to the Nezer Hakodesh, two questions remain: what is the radius of curvature about which the beams will be bent, and once that curvature is defined, is there a satisfactory lower bound for the radius of the fully elastic non-bendable rod. In the previous case we used Euler's solution for a rod that could support itself while vertically oriented without buckling under its own weight. Depending on the radius of curvature chosen, that may not be a practical bound, requiring a fallback to the minimum amount of material useful to the temple. Potential curvatures could come from comparisons to capabilities and condition of the donor's body, which Tosfos imply is the theme of this type of pledge, or possibly the shapes of Hebrew letters, some of which are referred to as bent or כפוף.

Conclusion

In this work, we applied concepts from structural mechanics to explain the Gemara's intent with regard to the pledges of קומתי and מלא קומתי which are otherwise left undefined by the Talmudic text. Though this question is fascinating, it is interesting to note that it did not make a huge mark on the Rambam. In *Hilchos Erechin* 2:6 the Rambam anonymously quotes the opinion of Rav Yehuda:

האומר קומתי עלי, כסף או זהב--נותן שרביט שאינו נכפף מלוא קומתו, ממין שפירש; אמר מלוא קומתי עלי--נותן אפילו שרביט הנכפף, ממין שפירש.

Aside from including the material specification in this halacha, the Rambam uses the words מלוא קומתו only to define the length of the unbending rod, leaving no doubt where he stands in the Rashi/R' Gershon/Nezer Hakodesh debate.

That the Rambam is ambivalent towards this question is not striking. What is, however, is what's missing from the last half of the previous halacha:

אמר משקל ידי, או רגלי עלי--רואין כמה היא ראויה לשקול, ונותן ממין שפירש.

Instead of following the exacting procedure outlined in the Mishna by R' Yehuda to evaluate the weight of an attached limb mentioned above, the Rambam says we should see how much the arm should weigh, רואין כמה היא ראויה לשקול. The Kessef Mishna concludes from these words that

the Rambam is opting for the rough estimations afforded by visual inspection which is the opinion of Rabi Yose, who is quoted on *Erechin* 19b as saying

א"ל ר' יוסי עד שאומדין ימודו את היד.

If one must estimate, he should estimate the value of the hand to begin with, in contradistinction to R' Yehuda, who advocates for exacting evaluations of limb measurement. When one considers the Rambam's overall approach to philosophy and halacha, we would have expected him to be a champion of R' Yehuda's position, *כמה דאפשר עבדינן*, that whatever is within the realm of possible we should do. Why should we not be as exacting in the evaluation of bone density and beam bending as we are with astronomical calculations, were the Rambam himself goes down to the fractions of a second?

One could suggest the reason for the Rambam's disinterest in this question is entirely practical, summed up in the final words of *Erechin* 2:7

לפי שבגדרים, הולכין אחר לשון בני אדם

Unlike the trajectories of the heavenly sphere, which were set in motion by The Creator, and which we are commanded to calculate with ever tighter precision to better serve Him and appreciate the greatness of God's Universe, the realm of vows and oaths, *Damim* in particular, is where the ball is placed squarely in Man's court. It is our opportunity within the larger halachic framework to set rules for ourselves that restrict or compel certain behaviors. Everything hinges not only on our human language, but on our thoughts and memories within. As the Mishna on Menachos 106b says

כסף פירשתי ואיני יודע מה פירשתי יהא מביא עד שיאמר לא לכך נתכוונתי.

When someone forgets the value of their silver pledge, they bring until they exclaim *לא לכך נתכוונתי* - so much I did not intend. The Rambam reuses this language liberally when discussing *Damim* pledges of unknown amounts, concluding his brief discussion of *Erechin* 19a with:

וכן נותן שרביט מלא קומתו, אפילו של עץ: הכול לפי ממונו, ודעתו.

If one failed to specify the material for *קומתו*, מלא קומתו, everything hinges on the available funds, but more importantly, the person's intention, something for which we have yet to develop the tools to accurately measure. ❧