Probabilistic Analysis of the Propagation of Latent Mamzerus

By: SIMEON M. BERMAN

As is well known, the *mamzer* is defined as the offspring of a man and a woman born as a result of a conjugal relation that is forbidden under the penalty of *koreis* (*Yevamos* 48a). The definition extends to an individual having at least one parent who is a *mamzer*, so that *mamzerus* is a hereditary characteristic. A *mamzer's* selection of a marriage partner is, by *halacha*, limited not only to other *mamzerim* but also to members of five other genealogical groups of Jews outside the mainstream (*Kiddushin* 69a). A person who knows that he is a *mamzer* but is not publicly known as such might be tempted to keep it as a secret in order to marry within the mainstream. Any offspring of such a marriage would also be *mamzerin* who might marry mainstream spouses and potentially produce more *mamzerim*. In this way unidentified *mamzerim* might spread the stigma to large sections of the Jewish community.

In *Yevamos* 78b the Gemara considers the problem of the hidden *mamzer*. R. Eliezer states that "I would absolve anybody of *mamzerus* even at the third generation." Rashi infers from this that R. Eliezer holds that it is impossible for the family line of an individual *mamzer* to survive for more than two generations. The Gemara expounds on R. Eliezer's statement in the cases of publicly known and publicly unknown *mamzerus*, as follows:

> A publicly known *mamzer* will not find a mainstream spouse. (Even if he marries within his restricted group, any of his children will also not find a mainstream spouse.)

Simeon M. Berman is Emeritus Professor of Mathematics at the Courant Institute of Mathematical Sciences of NYU, where he was on the faculty for 45 years, before retiring in 2010. He is the author of over one hundred ten papers published in mathematical research journals, three textbooks, and a research monograph. His specialties are probability theory and mathematical statistics. He served as the principal investigator of mathematical research grants awarded by the National Science Foundation, the U. S. Army Research Office, and the National Institutes of Health. He was awarded a Guggenheim Fellowship for the academic year 1992-1993.

> A *mamzer* whose status is publicly unknown will die at the hand of Heaven in order to prevent his marriage to a mainstream spouse.
>
> A *mamzer* whose status is "known but not known" will not have a family line beyond the third generation. (At that point the doubtful *mamzerus* is likely to be forgotten, and forbidden marriages might take place.)

Our focus is on the case of a *mamzer* whose status is unknown to the public and, as a consequence, will die at the hand of Heaven. The Gemara does not take up the fact that this class of *mamzerim* is actually composed of two distinct subsets. The first consists of those who are aware of their status but keep it as a secret in order to find a marriage partner in the Jewish mainstream. The complementary subset consists of *mamzerim* who are not aware of their status and consequently might unintentionally marry a mainstream spouse. We refer to such a person as a "latent *mamzer*." Such a situation can arise when a man and a woman marry but they and the public are unaware that their union is forbidden. Although the couple is not condemned for the unintentional transgression (*onus*), the offspring are still *mamzerim*, and, in the absence of public disclosure, are latent *mamzerim*. As "innocent bystanders" their fate should be no worse than that of their parents. Since this issue is not discussed in *Yevamos* 78b, it is reasonable to infer that the resulting conclusion about the unknown *mamzer* does not apply to the latent *mamzer*.

An illustration of the latent *mamzer* case is given in *Yevamos* 83b. A woman whose husband went overseas and was not heard from, remarried on the basis of the testimony of two witnesses who reported that the missing husband was dead. However, the report was mistaken and the husband was actually alive and returned. The offspring of the second marriage were deemed *mamzerim*. Even if the first husband was alive and did not return and was publicly believed to be dead, the offspring would still be considered theoretically to be *mamzerim—latent mamzerim*.

The general rule that applies to the case of the latent *mamzer* is summarized by Rav Moshe Feinstein in *Iggeros Moshe, Even HaEzer, Chelek 4,* Responsum 9:

An *erva* offspring is a *mamzer* even if there is no intentional transgression in the marriage of the parents. … Identification as a *mamzer* is not a punishment for a sin (of the parents) but is a consequence of simply being the offspring of a forbidden marriage.

Suppose that G-d actually spares the latent *mamzerim*, with the result that they marry and produce children who are also latent *mamzerim*. In this way the stigma might spread to large sections of the community. The purpose of this paper is to show how G-d can remotely control and eliminate such *mamzerus* without resorting to the death of the latent *mamzer*. Upon a recent review of this discussion in *Yevamos*, it occurred to this writer, as a mathematician specializing in probability, that there are theoretical tools in this area that can furnish a method of calculating the probability of the spontaneous extinction of the family line of a *mamzer* independently of halachic considerations. This area in probability is known as the theory of "branching processes," introduced by the famous mathematician and geneticist Francis Galton (1822–1921). He investigated the following problem. Suppose a male individual adopts a new surname for himself, and then has children. In Galton's society in his time the surname was determined by that of the father and so the male children inherited the surname. In this way the male descendants transmitted the surname down through the generations as long as there was no interruption in the birth of males. The name became extinct as soon as there was a generation without males. The process of the transmission of the surname over the successive generations has been called the "branching process" because each child in the family line who bears the name transmits it to his offspring and initiates a new branch. Thus the transmission of the family name is analogous to that of the transmission of *mamzerus*. Galton based his theory on the hypothesis that there was a particular random mechanism that defined the transmission of the surname down through the generations and then, by means of probability theory, calculated the probability of extinction at any specified generation and the probability of ultimate extinction. His calculations extend directly to our problem by simply replacing his "surname" by our term "*mamzerus*." We note a formal but inessential difference between the propagation of the surname in Galton's branching process and the propagation of *mamzerus*. In the former case the surname is transmitted only by male offspring, so that females are not counted as offspring. In the latter case both males and females are counted as offspring. The mathematical analogy between the two cases is not affected by this distinction.

Lotteries in *Tanach*

We apply Galton's theory to describe how the family line of a *mamzer* can become ultimately extinct under ordinary conditions. The theory is based on the hypothesis that the numbers of offspring in successive generations descended from a *mamzer* are the results of the outcomes of repeated plays of a game of chance. In particular, we will consider a game in the form of a lottery, where balls labeled with numbers are repeatedly drawn at random from an urn. On the one hand, by the nature of such games, the resulting outcomes are humanly unpredictable. On the other hand, the belief in Divine omnipotence and omniscience implies that G-d actually determines the outcome of each play of the game and can predict all future outcomes. The mathematical theory underlying such a game provides probabilities of outcomes without regard to G-d's designs.

There is an extensive literature on the subject of the role of chance in the way G-d runs the universe and its relation to human free will. Recent articles on this subject in the Orthodox Jewish literature are those of Alan Kadish [*Ḥakirah* 20 (2015) 115–132)] and Nathan Aviezer [*Ḥakirah* 21 (2016) 61–68] who discuss this issue from the point of view of quantum mechanics. Here we make no attempt to discuss this large issue and simply describe a theory of the elimination of *mamzerus* based on humanly unpredictable random outcomes.

There are major examples of incidents described in *Tanach* in which G-d's will concerning the fate of individuals and communities is made known as results of random outcomes, particularly the results of the drawing of lots. While the Gemara (*Sanhedrin 17a*) describes the lottery that was used to select the seventy elders (*Bemidbar 11*), the first explicit reference in *Tanach* to a lottery is in the assignment of the portions of the land of Israel to the twelve tribes (*Bemidbar* 33). The purpose of the use of the lottery was to give the appearance of fairness in order to avoid complaints over the distribution. Indeed, according to the commentary *Kli Yakar* on sentence 54 of that chapter, the particular issue that could be potentially controversial was the commandment to destroy all idols found in the land of each tribe and expel those who worshipped the idols. As a consequence, each tribe might like to get a portion with a small number of idol worshippers in order to minimize the resources required for their elimination. As recorded in the book of Joshua (chapters 13–18), five of the twelve tribes actually requested and were granted particular portions of the land before the drawing of lots so that it would appear that only the seven remaining tribes participated in the lottery, contrary to the commandment to Moshe. This issue is resolved in the Gemara (*Bava Basra* 122a) where it is stated that all twelve tribes actually drew lots. The Gemara states that Yehoshua and Eleazar stood with a pair of boxes, one containing tickets with the names of the tribes and the other containing tickets with the title of the portion of the land. The tickets were drawn in successive pairs, one from each box, resulting in an assignment of the portion of land to the named tribe. Before the drawing of each pair, the *Urim V-Tumim* on the breastplate of Eleazar would indicate the outcome of the drawing and the resulting assignment of land to the tribe, and this was confirmed by the actually observed outcome. In particular, the tribes that had requested specific portions miraculously received them. There was almost no chance that the matching of the twelve tribes to the lands actually assigned by G-d was a coincidence. Indeed, the probability of twelve correct matches in pairs of tickets drawn at random is one out of 4.79×10^8, or close to one out of one-half billion.

Two other major lotteries involved the public identification of individuals who had sinned in secrecy. As recorded in Joshua, chapter 7, *Achan* violated G-d's commandment and secretly stole some of the property taken from the destruction of Jericho, and as a result, Israel's army was defeated in the battle against Ai. As the result of a public lottery he was identified as the sinner who was responsible for the defeat. The second example is that of the identification of Jonah as the passenger on the ship that was in danger of being wrecked because of his secret flight to avoid G-d's command to go to Nineveh and warn the people of their impending doom if they did not repent. Other examples in *Tanach* of the drawing of lots are relatively minor because the outcomes were not of major religious significance:

i) The choice of the goat to be sent to *Azazel* on Yom Kippur (Leviticus 16),
ii) The assignment of the people to live in Jerusalem after the rebuilding of the *Beis HaMikdash* (Nehemiah 11), and
iii) The arrangement of the tasks of the *Kohanim* (Chronicles I, chapters 24, 26).

In all these examples the lottery is a device that is used by G-d, where the outcome is known to Him beforehand but not to the participants.

The Branching Process: A Sequence of Lottery Drawings

Consider an urn containing balls of identical sizes but where each ball belongs to some subset, each defined by a particular characteristic such as the color of the individual ball; for example, subsets consisting of balls colored red, white, blue, etc., respectively. We say that a ball is drawn "at random" if each ball is assumed to have the same probability of being drawn. We define the probability of drawing a ball of a specific color as the proportion of balls of that color in the urn. For example, if ten percent of the balls in the urn are red, then the probability that a ball drawn at random is red is 1/10.

Suppose next that the subsets of balls are distinguished not by color but by integers 0, 1, 2, …. We describe the following game that determines the number of offspring of an individual. A person draws a ball at random from the urn and the number that turns up on that ball represents the number of his offspring. We call this number the "size of the first generation." If the number is 0 then there are no offspring, and we say that there is "extinction" at the first generation. If the ball that is drawn has a number greater than 0, then the first generation has that number of members, and each one independently produces offspring in exactly the same way as their parent, namely, by drawing balls at random and observing the numbers that are drawn. It is assumed that the balls that are drawn are immediately replaced after each drawing so that the proportions and corresponding probabilities remain unchanged.

The second generation consists of the offspring of the first generation. If none of the members of the first generation have offspring, then we say that there is extinction at the second generation. On the other hand, if there are offspring, these form the second generation and the "size of the second generation" is the sum of the numbers of offspring of the first generation. The third generation consists of the offspring of the second generation, and is obtained from it in the same way that the second is obtained from the first, namely, each member independently draws a ball from the urn and produces the number of offspring indicated on the ball. The size of the third generation is defined as the sum of the numbers of offspring of the second generation. We say that there is extinction at the third generation if there are members of the second generation but none have offspring.

This process of creating successive generations of offspring continues either indefinitely or until there is extinction at some generation. If extinction occurs at a particular generation then that generation has no members and so there are no real future generations. However, for the purpose of consistency of the mathematical description, we define future theoretical generations as those following the generation where extinction actually occurs; for example, if extinction occurs at the third generation then the fourth and all subsequence generations are theoretical and have no members. It follows that if a generation, theoretical or real, has no members then extinction must have occurred at that generation or some previous one; for example, if the fourth generation has no members then extinction must have occurred at either the first, second, third, or fourth generations. The result is:

> A given generation has no members if and only if extinction occurs at or before that generation.

The likelihood of extinction is influenced by the composition of the labeled balls in an urn: An urn with a larger proportion of balls with large numerical labels will tend to yield smaller probabilities of extinction than an urn with a smaller proportion of such balls because small numbers signify fewer offspring. We define the *birth rate* for a given urn as follows:

Birth rate = Average of the label numbers of the balls in the urn.

(The reader with a knowledge of elementary probability will recognize this as the expected number of offspring for each individual.) This quantity plays a central role in the theory of branching processes.

Three major results of Galton are:

1. There are relatively simple procedures for calculating the probability of extinction according to the generations, and the probability of ultimate extinction.
2. If the birth rate for the urn is less than or equal to 1, then the probability of ultimate extinction is 1, that is, is certain to occur. An exception to this rule occurs in the trivial case where all the balls have the common label 1, so that each individual always has exactly one offspring and every generation has exactly one member, which implies that there never is any extinction.
3. If the birth rate is greater than 1, then the probability of ultimate extinction is a number greater than 0 and less than 1, which implies that ultimate extinction is possible but not certain. An exception to this rule occurs when none of the balls in the urn has the label 0, so that every member is certain to have at least one offspring and so there never is any extinction.

These results are recorded in the book of William Feller, *An Introduction to Probability Theory and its Applications*, Vol. 1, Third Edition, pp. 293–298, John Wiley and Sons 1968.

As described above, the branching process is a game consisting of a sequence of random drawings of balls from an urn, where each ball has a label identified as one of the integers 0, 1, 2, …. The probability of drawing a ball having a specified numerical label is equal to the proportion of balls in the urn having that label. The particular number arising from a draw represents the number of offspring of an individual. The drawings are done in consecutive groups defined as generations, and the size of a given generation is the sum of the numbers drawn in the previous generation. Extinction occurs as soon as some generation is of size 0.

The following remark is meant for the reader with a knowledge of probability, who may reasonably ask why the author chose the particular probability model of lottery drawings to generate the distribution of the number of offspring. Indeed, the discussion could have been streamlined by simply introducing an "integer-valued random variable." The answer is that since most of the readers of this paper will not have a knowledge of basic probability, the process has been described in the concrete terms of the lottery, rather than in the abstract terms of probability theory.

Calculation of the Extinction Probabilities by Generation: A Numerical Illustration.

In this section we present numerical illustrations of the extinction probabilities in four cases, corresponding to four different proportions of balls of integer-valued labels, and where there are three labels 0, 1, and 2, corresponding to 0, 1, and 2 offspring, respectively. Table 1 displays the probabilities of extinction at or before generations 1 through 10 in the two situations where the proportions of balls with labels 0, 1, 2 are (.6 .2 .2) and (.4 .4 .2), respectively. The birth rate for the first urn is

$0 \times .6 + 1 \times .2 + 2 \times .2 = .6.$

Similarly, the birth rate for the second urn is .8. In these two cases the probability of ultimate extinction is 1. Table 2 displays similar probabilities in the two cases where the proportions are (.2 .4 .4) and (.2 .3 .5), respectively. The probabilities of ultimate extinction are .5 and .4, respectively.

Generation	**Proportions (.6 .2 .2) Birth rate .6**	**Proportions (.4 .4 .2) Birth rate .8**
1	.792	.592
2	.884	.707
3	.933	.783
4	.961	.836
5	.977	.874
6	.986	.902
7	.992	.924
8	.995	.940
9	.998	.953
10	.999	.963

Table 1. Extinction Probabilities by Generation for Proportions (.6 .2 .2) and (.4 .4 .2).

Note the comparison of the implication of R. Eliezer's statement that there are no *mamzerim* after the second generation with the probabilities of extinction at or before the third generation, namely, .933 and .783. (These are the probabilities that there are no *mamzerim* in the third generation.)

Generation	Proportions (.2 .4 .4) Birth rate 1.2	Proportions (.2 .3 .5) Birth rate 1.3
1	.296	.280
2	.353	.323
3	.391	.349
4	.418	.366
5	.437	.377
6	.451	.384
7	.462	.389
8	.470	.392
9	.476	.394
10	.481	.396
Probabilities of ultimate extinction	0.5	0.4

Table 2. Extinction Probabilities by Generation for Proportions (.2 .4 .4) and (.2 .3 .5)

As indicated in Table 2, the probabilities of no offspring after the second generation are .391 and .349, respectively, and the probabilities of ultimate extinction are 0.5 and 0.4, respectively. By contrast, ultimate extinction is certain to occur when the birth rate is less than or equal to 1, as in Table 1. This raises the question: If, as in the examples in Table 2, where the birth rate is greater than 1, the probabilities of ultimate extinction are only 0.5 and 0.4, how can we be sure that latent *mamzerus* will be eliminated in such cases? The answer is implied by the outcomes of the three major lotteries cited above, namely, those that determined the assignment of the portions of the land distributed to the twelve tribes, and those that determined the fates of Achan and Jonah. In each of these lotteries the prior probability of "winning" was extremely small, and yet the outcome that was Divinely desired was realized.

Conclusion

In this work it is shown that an established mathematical theory can illuminate the discussion of a question suggested by, but not considered, in the Gemara. In *Yevamos* 78b there arises the question of how *mamzerus* is prevented from transmission to future generations. The Gemara is explicit about the Divine action that limits the transmission of *mamzerus* to at most three generations. However, the case where the *mamzer* is not aware of his status—the latent *mamzer*—is not considered separately from that of other *mamzerim*; indeed, it would seem that the fate of the latent *mamzer* should not be as harsh as that of the self-aware *mamzer*, so that the restrictions on the offspring of the latent *mamzer* should not be as severe as those of the self-aware *mamzer*. Hence there arises the question of how latent *mamzerus* is prevented from extensive transmission. The theory of branching processes provides the probability that any hereditary characteristic, including the present example of *mamzerus*, will become spontaneously extinct after a specified number of generations. In particular, it is shown that if the birth rate is less than or equal to 1, then, with a trivial exceptional case, ultimate extinction is certain to occur. If the birth rate is greater than 1, then, with a trivial exceptional case, the probability of ultimate extinction is between 0 and 1.

It has been the belief that G-d has His own way, unknown to man, of eliminating *mamzerus*. The current work is an attempt to suggest a theory, in the framework of probability, of how this Divine strategy might possibly be executed. ☙