Rambam and the Size of the Sun

By: YORAM BOGACZ

The feat of measuring the size of the Earth was first accomplished by Eratosthenes, born around 276 B.C.E. in Cyrene, in modern-day Libya. For many years, Eratosthenes was the chief librarian at Alexandria, possibly the most prestigious academic post of the ancient world. Eratosthenes had learned of a well situated near the town of Syene in Southern Egypt, close to modern-day Aswan. At noon around 21st June each year (the summer solstice), the Sun was directly overhead and shone straight into the well. In Alexandria, several hundred kilometres to the north, this never happened. Eratosthenes thought that he could exploit this fact to measure the circumference of the Earth.

Figure 1 shows how parallel rays of light from the Sun hit the Earth at noon on 21st June. At noon—the same moment that some of the rays were plunging into the well at Syene—Eratosthenes stuck a stick vertically in the ground at Alexandria and measured the angle between the Sun's rays and the stick. This angle is equal to the angle between two lines drawn from Alexandria and Syene to the centre of the Earth. He measured the angle to be 7.2°.

Eratosthenes took the Earth to be perfectly spherical.[1] So if the angle between Syene and Alexandria is 7.2°, then the distance between the two cities represents $\frac{7.2}{360}$ of Earth's circumference, because there are 360 degrees in a full circle. So the distance between the two cities represents one-fiftieth of the circumference of the Earth [$\frac{7.2}{360} = 0.02 = \frac{1}{50}$].

The next step was to measure the distance between the two towns. It is not clear how Eratosthenes did this. One suggestion is that he measured the circumference of a cart wheel, and recorded the number of revolutions made by the wheel on the journey between Alexandria and Syene. The distance turned out to be 5,000 stades. Since this represents $\frac{1}{50}$ of the Earth's circumference, the total circumference must be 250,000 stades.

1 This is correct to a first approximation. The Earth is actually an oblate spheroid, meaning that it bulges slightly at the equator, due to its rotation around its axis. So the equatorial radius is 6378 km, while the polar radius is 6356 km.

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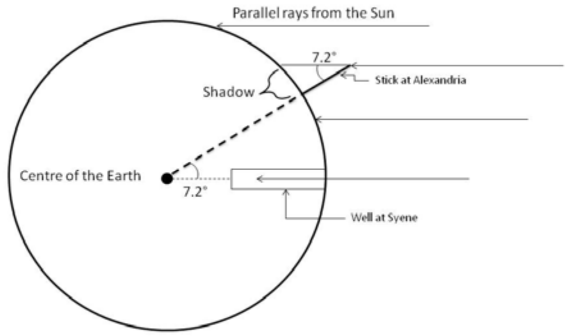

Figure 1. Eratosthenes used the shadow cast by a stick at Alexandria to calculate the circumference of the Earth. He concluded the experiment on the day of the summer solstice, where the noon Sun is directly above the Tropic of Cancer. Diagram is not to scale.

How far is 250,000 stades? The stade was a standard distance over which races were held. The Egyptian stade (as opposed to the Olympic stade) was 157 metres, which gives a circumference of 39,250 km. The modern figure is 40,075 km, a difference of just 2%. There were several inaccuracies in the method used by Eratosthenes. For example, Syene is not directly south of Alexandria. Furthermore, it is situated not precisely on the Tropic of Cancer, but slightly north of it (see Map 1). Nonetheless, given the tools at his disposal, Eratosthenes obtained a result of astonishing accuracy.

Map 1. Alexandria (top left) is on the shore of the Mediterranean Sea. The modern city of Aswan, close to the ancient Syene, is just north of Lake Nasser (bottom right). Notice that Aswan is not directly south of Alexandria; nor is it on the Tropic of Cancer.

The next step was to deduce the size of the Moon and Sun. The groundwork had been laid by earlier thinkers, but the calculations were incomplete until the size of the Earth had been established. Eratosthenes compared the size of Earth's shadow cast upon the Moon during a lunar eclipse, as shown in figure 2.

It was now possible to deduce that the Moon's diameter was about one-quarter of the Earth's. Once Eratosthenes had shown that the Earth's circumference is about 40,000 km, its diameter is simply this figure divided by π (*pi*, the ratio of a circle's circumference to its diameter), yielding roughly 12,700 km (modern figure: 12,756 km). Therefore, the Moon's diameter was one-quarter this figure, or nearly 3,200 km.

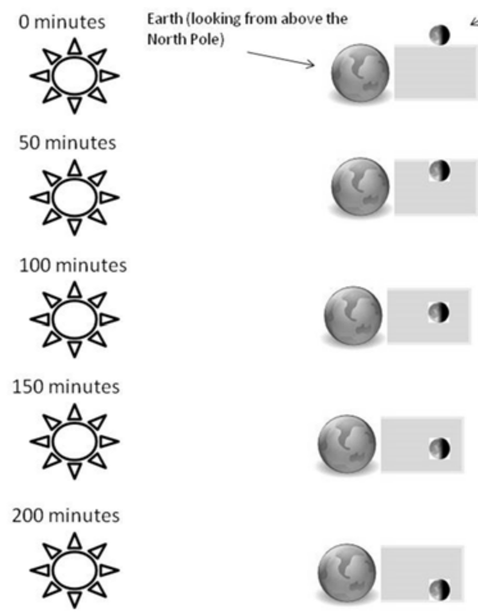

Figure 2. The relative sizes of the Earth and Moon can be estimated by observing the Moon's passage through Earth's shadow during a lunar eclipse. The Earth and the Moon are very far from the Sun, so the size of the Earth's shadow is much the same as the size of Earth itself. The diagram shows the moon passing through Earth's shadow. In this eclipse, when the Moon passes roughly through the centre of Earth's shadow, it takes 50 minutes for the Moon to go from touching the shadow to being fully covered, so 50 minutes is an indication of the moon's own diameter. The time required for the front of the Moon to cross the entire shadow is 200 minutes, so Earth's diameter is about 4 times that of the Moon.

It was then easy for Eratosthenes to estimate the distance to the Moon. One way to do this is to stare at the full Moon, close one eye and stretch out your arm. You will notice that you can cover the Moon with the end of your forefinger. Figure 3 shows that your fingernail forms a triangle with your eye. The Moon forms a similar triangle, with a vastly greater *size* but identical *proportions*. The ratio between the length of your arm and the height of your fingernail, about 100:1, is the same as the ratio between the distance to the Moon and the Moon's own diameter. This means that the distance to the Moon is roughly 100 times greater than its diameter, yielding a distance of 320,000 km (current figure: about 384,000 km).

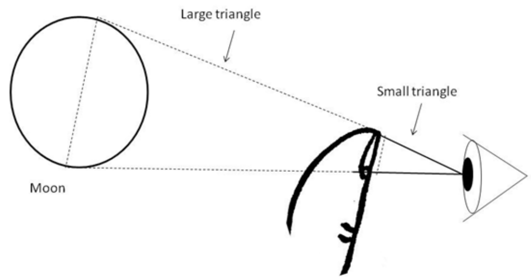

Figure 3. Having estimated the size of the Moon, it is quite easy to calculate the distance to the Moon. Notice that you can block out the Moon with a fingertip at arm's length. The ratio of a fingernail's height to an arm's length is roughly the same as the ratio of the Moon's diameter to its distance from the Earth. An arm's length is about 100 times longer that a fingernail, so the distance to the Moon is about 100 times its diameter.

Next, Eratosthenes built upon the arguments of Aristarchus of Samos (circa 310–230 B.C.E.) If moonshine is reflected sunshine, Aristarchus explained, then the half-Moon must occur when the Sun, Moon and Earth form a right-angled triangle, as shown in Figure 4.

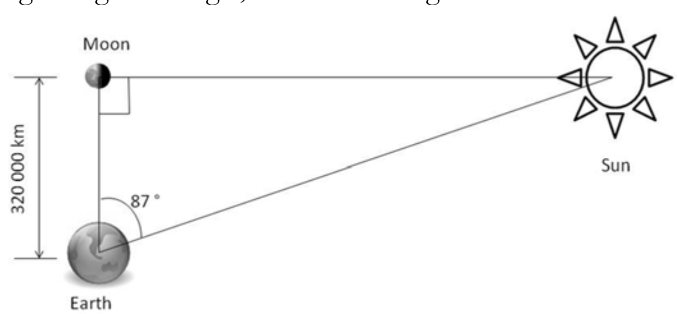

Figure 4. Aristarchus argued that it is possible to estimate the distance to the Sun by using the fact that the Earth, Moon and Sun form a right-angled triangle when the Moon is at half phase (quadrature). Diagram is not to scale.

Aristarchus measured the angle at the Earth as 87°. Using elementary trigonometry (which is explained at the end of this article), he calculated that the Sun must be about 20 times as far away from the Earth as is the Moon. In reality, the precise moment of *quadrature* (when the Sun, Earth and Moon form a right-angled triangle) is difficult to determine, and Aristarchus' measurement of the angle was limited by his instruments. At quadrature, the angle at the Earth is 89.85°; the Sun is thus about 400 times farther away than the Moon.

Finally, deducing the size of the Sun was easy. During a total solar eclipse, the Moon fits almost perfectly over the Sun. This means that the ratio of the Sun's diameter to the Sun's distance from the Earth must be the same as the ratio of the Moon's diameter to the Moon's distance from the Earth, as shown in Figure 5.

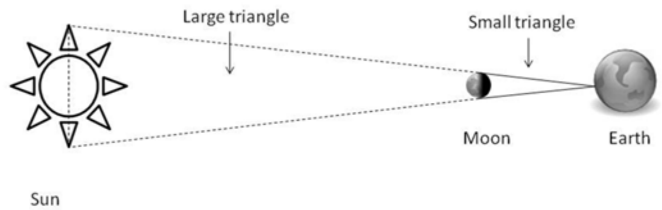

Figure 5. It is possible to estimate the size of the Sun, once its distance is known. A total solar eclipse occurs when the Moon is exactly between the Sun and the Earth. The Moon appears to exactly cover the Sun. The eclipse observer is at the apex of two similar triangles (i.e., possessing identical proportions). Knowing the distance to the Moon and to the Sun and knowing the diameter of the Moon is enough to deduce the diameter of the Sun.

Thus, Eratosthenes concluded that the Sun was about twenty times farther away from the Earth as the Moon, and that its diameter was about 5½ times that of Earth. Since the volume of a sphere is proportional to the cube of the diameter, this implied that the Sun's volume is $(5½)^3 = 166\frac{3}{8}$ times greater than the volume of Earth.

The results described above were well known in the ancient world, and were incorporated by Ptolemy into his monumental *Almagest*. Later, his work was translated into Arabic and studied intensively by Arab scholars during the golden age of Arabic science. This was the great flowering of science (approximately 9^{th}–12^{th} centuries C.E.), particularly astronomy, in the Arab world, while science languished in Medieval Europe.

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Rambam learnt his astronomy from contemporaneous Muslim scholars. He writes in *Moreh Nevuchim* that:

> Ibn Aflach, with whose son I associated, composed a famous book about this subject. Later, the superb philosopher Abû Bakr ibn al-Sâ'igh got involved in this matter. I studied with one of his students, and he taught me the relevant proofs, and I copied them from there...[2]

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2 רמב"ם, מורה נבוכים חלק ב פרק ט: ...וכבר חבר בזה אבן אפלח האשביליי אשר התחברתי עם בנו, ספר מפורסם, אחר כך השתדל בזה הענין הפילוסוף המעולה אבובכ"ר בן אלצאי"ג אשר קראתי עם אחד מתלמידיו והראה אפני ראיות, כבר העתקנום ממנו...

Furthermore, Rambam obtained many astronomical values from the Arab astronomer Al-Battani (circa 858–929 C.E.). This outstanding astronomer and mathematician whose full name—Abu abd Allah Muhammad ibn Jabir Sinan Al-battani Al harrani As-sabi—was mercifully Latinized as Albategni and Anglicised as Al-Battani, was the best known of the Arab astronomers in Europe during the Middle Ages. His principal written work, a compendium of astronomical tables, was translated into Latin in about 1116, about two decades before the birth of Rambam.[3] That Rambam relied on Al-Battani's numbers was pointed out by as early an authority as *The Commentator* (המפרש). This is a reference to Rabbi Ovadiah ben David (d. 1341 C.E.), whose commentary to Rambam's astronomical treatise has been printed in all editions of Rambam's work for centuries, thereby earning him the status of *the* Commentator. He writes that "Rambam relied on Al-Battani and nobody else."[4] In the 20th century, Rambam's work was closely analysed by Professor Otto Neugebauer (1899–1990), who was one of the most distinguished of the archaeo-astronomers (historians who research ancient astronomy).[5] His comparison of Al-Battani's figures with Rambam's numbers indicates clearly that Rambam used these—the best available figures at the time—for his calculations. This is corroborated by contemporary researchers such as J.J. Ajdler.[6]

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Rambam, in his *Introduction to the Mishna*, makes the point that something that appears counterintuitive can still be true. This is not a trivial statement. Rambam argues that a rigorous logical argument is sufficient to

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3 Britannica writes that Al-Battani was an "Arab astronomer and mathematician who refined existing values for the length of the year and of the seasons, for the annual precession of the equinoxes, and for the inclination of the ecliptic. He showed that the position of the Sun's apogee, or farthest point from the Earth, is variable and that annular (central but incomplete) eclipses of the Sun are possible. He improved Ptolemy's astronomical calculations by replacing geometrical methods with trigonometry. From 877 he carried out many years of remarkably accurate observations at ar-Raqqah in Syria."
"Battānī, al-," *Encyclopedia Britannica: Ultimate Reference Suite* (Chicago: Encyclopedia Britannica, 2009).

4 פירוש רבי עובדיה בן דוד על הלכות קידוש החודש (פרק יב, הלכה א): הוא ז"ל סמך על דעת אלבתני, לא על זולתו.

5 O. Neugebauer, "The Astronomy of Rambam and its Sources," *HUCA*, vol. 22, 1949, pp. 321–363.

6 י. י. איידלר, הלכות קידוש החודש ע"פ הרמב"ם, (תשנ"ו) עמוד 122 ועמודים 228-233.

convince us of the veracity of something that appears to be untrue, even preposterous. The Sun appears to be a small disk in the sky but, argues Rambam, it is actually bigger than the Earth, and we can deduce its size and distance from Earth even though these parameters are not directly accessible to measurement. He proceeds to explain this, and in the process explicitly credits Ptolemy's *Almagest*.[7] He writes that the Sun's size is $166\frac{3}{8}$ times the size of Earth.[8] From the foregoing discussion, it is obvious that

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7 רמב"ם, הקדמה לפירוש המשניות: ואם יתלמד בספר ההנדסה וידע מה שצריך מן היחסים בצורות הכדוריות וזולתן, ואחר כן יעתק אל הספר המחובר בזה הענין והדומה לו, רצוני לומר ספר התכונה הידוע בשם ספר אלמגסטי, אז יתאמת אצלו זה הדבר, ויהפוך אצלו למאמר אמיתי שאין ספק בו, שכבר נתקימה עליו ראיה ברורה.

8 **שם:** ואני אתן לך בזה משל מבואר, והוא, נניח שאמרנו לאיש מאנשי החכמה ברפואה ובחשבון ובחיבור הניגונים, בקי בטבעיות, חריף השכל, טוב הטבע, והוא ערום מחכמת ההנדסה וחכמת התכונה: 'מה תאמר באיש הטוען כי גרם השמש, זה אשר נראהו כעגול קטן, הוא גוף כדורי, **וגודל אותו הכדור מאה וששים ושש פעם ושלוש שמיניות פעם כגודל כדור הארץ**? ושכדור הארץ אשר בו שערנו, הוא כדור שהיקפו עשרים וארבעה אלף מיל? ויהיה אפשר לפי זה להגיע אל הידיעה, כמה מילין יש במידת גדלו של כדור השמש?' הרי אין ספק שאותו חריף השכל המיטיב לדעת מן החכמות מה שזכרנו, לא ימצא בנפשו מקום בשום פנים לאמת זאת הטענה, ויהיה כל זה אצלו דבר בלתי מושג...

Notice that Rambam writes that Earth's circumference is 24,000 *mil*. A *mil*, in rabbinic writings, is equivalent to 2,000 cubits. Taking a cubit as approximately 50 cm, this yields 12,000 km for Earth's circumference. Recall that Eratosthenes calculated Earth's circumference as about 40,000 km. I wrote about this to Rabbi Zalman Menachem Koren:

בס"ד
לכבוד הרה"ג ז. מ. קורן שליט"א,

עברתי שוב על המסמך שכבוד-תורתו שלח לי לפני כמעט שנה (מצורף). ישנה נקודה אחת שבה התקשיתי. הרמב"ם מסתמך על ספר האלמגסטי, ולכן הוא כותב שגודל (=נפח) השמש הוא 166 ושלוש-שמיניות מגודל כדור הארץ. אך אם הוא סמך על האלמגסטי, ודברי האלמגסטי נשענים על שיטתו ותוצאותיו של Eratosthenes, מדוע כותב הרמב"ם שהיקף כדור הארץ הוא 24 אלף מיל? הרי ארטוסטנס הגיע למסקנה שהיקפו של כדור הארץ הוא כ- 40000 קילומטר?!

בכבוד רב,

יורם בוגץ'
יוהנסבורג, דרום אפריקה

Here is Rabbi Koren's response:

בס"ד
לכבוד
הרב יורם בוגץ'

ידידי המהנדס ר' יעקב גרשון וייס עסק בעניין באריכות בספרו מדות ומשקלות של תורה עמ' רצ"ד, ועוד קודם לכן במאמר שפרסם בקובץ 'תורה ומדע' שבט תשל"ז. לדבריו שם מידה זו

Rambam meant that the Sun's *volume* is 166 times greater than Earth's *volume*. He did *not* mean that the Sun's diameter is 166 times the diameter of Earth. Likewise, Rabbi Judah HaLevy, in his classic *Kuzari*, writes that the Sun's size is 166 times the size of the Earth.[9]

Conclusion

We know today that the Sun is about 400 times farther away from us than the Moon; we also know that its diameter is 109 times the diameter of Earth, making its volume 1.3 *million* times the volume of Earth. When saying that the Sun is 166 times as large as the Earth, Rambam was in error by several orders of magnitude. But what is the significance of this?

The methods and reasoning of Aristarchus and Eratosthenes were correct and sophisticated. Only their numerical results, limited by the available technology, were faulty. Furthermore, Rambam's overall point is valid: rigorous analysis can be trusted, even in the face of counterintuitive notions. The Sun is indeed much bigger than Earth, and we can reach that conclusion on the basis of careful argumentation and measurement.

של 24000 מיל נקוטה בתקופה ההיא על ידי גיאוגרפים ערביים. אלא שהמיל שלהם הוא בשיעור של 3000 אמה.

למעשה מובן שארטוסטנס לא דיבר על ק"מ, אלא שיער את השיעור בסטדיות. שיעור הסטדיה הוא אחד חלקי שבעה וחצי של המיל. אלא שהמיל הרומי היה בשיעור של 1400 מטר כפי שניתן להוכיח ממצאים שונים, וזה כמובן יותר מאלפים אמה לפי כל שיעור שהוא. וכך הסטדיה או כפי שהיא נקראת אצל חז"ל "ריס" היא כ186 מטר. אך אצל חז"ל כשנזכרת מידת ריס במסכת יומא פ"ו מ"ד, המיל (שהיא מילה רומית) הוא מושג מושאל והוא מומר לאלפיים אמה, וכך גם הריס מגויר. (אגב קיימות בקדמונים מסירות שונות הנוגעות למידה.ובספר מדות ומשקלות שם בעמ' ש בהע' 8 ציין במפורש שבספר אלמגסטי מצוין שמידת היקף הארץ היא 180000 סטדיות, כלומר 24000 מיל)

מאמר נוסף של ר' יעקב גרשון וייס (יחד עם ר' טוביה כאץ) שנוגע בעניין הוא מאמר שפרסמו במוריה טבת תשנ"ו (המאמר עוסק בעלות השחר שלשיטת הרמב"ם שגם הוא קשור לשיעור היקף הארץ ומשמעות המילה מיל בלשונו שם).

הספר וגם הקבצים התורניים נמצאים במאגר אוצר החכמה.

בברכה

זלמן מ. קורן

9 **ספר הכוזרי מאמר שלישי, מט: ...** אמנם צריך שתעיין בשורשים מן המקובל והכתוב וההקשות הנוהגות על הסדר המקובל להשיב התולדות אל האבות. ומה שיוציאך אליו, האמן בו, ו[גם] אם יהיה רחוק אצל סברתך ומחשבתך, כאשר תרחיק המחשבה והסברה העדר הרקות, וההקשה, השכלית מחייבת זה. וכאשר תרחיק הסברה שיכול להתחלק הגוף לאין תכלית, וההקשה השכלית מחייבת זה. **וכאשר תרחיק המחשבה שהארץ כדורית, ושהיא חלק אחד ממאה וששים ושש פעמים מעגול השמש**, וכל מה שיש במופתי התכונה, ממה שתרחיקהו המחשבה.

Addendum: Trigonometry Argument used by Aristarchus

We denote the Sun, Earth and Moon as S, E, and M respectively. Then the Sun-Moon distance is SM; the Sun-Earth distance is SE; and the Earth-Moon distance is EM.

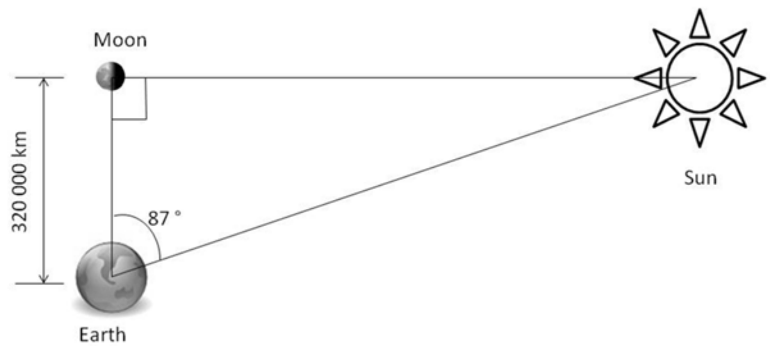

Figure 4. Aristarchus argued that it is possible to estimate the distance to the Sun by using the fact that the Earth, Moon and Sun form a right-angled triangle when the Moon is at half phase (quadrature). Diagram is not to scale.

In the right-angled triangle SEM, $\cos 87° = \frac{EM}{SE}$.

Therefore, $SE = \frac{EM}{\cos 87°} = \frac{320\ 000}{\cos 87°} = 6\ 114\ 343\ km$

The ratio EM: SE is $\frac{6\ 114\ 343}{320\ 000} = 19$

Substituting modern values for the parameters, we get

$$SE = \frac{EM}{\cos 89.85°} = \frac{384\ 000}{\cos 89.85°} = 146\ 677\ 363\ km$$

Therefore, the ratio EM: SE is $\frac{146\ 677\ 363}{384\ 000} = 382$